Transformation and Magnification by Simple Lenses

It is clear from the previous discussion that Gaussian beams transform in an unorthodox manner. Siegman uses matrix transformations to treat the general problem of Gaussian beam propagation with lenses and mirrors. A less rigorous, but in many ways more insightful, approach to this problem was developed by Self (S. A. Self, "Focusing of Spherical Gaussian Beams"). Self shows a method to model transformations of a laser beam through simple optics, under paraxial conditions, by calculating the Rayleigh range and beam waist location following each individual optical element. These parameters are calculated using a formula analogous to the well-known standard lens-maker's formula.

The standard lens equation is written as

$$\frac{1}{s/f}+\frac{1}{s''/f}=1. \qquad (2.12)$$

where s is the object distance, s'' is the image distance, and f is the focal length of the lens. For Gaussian beams, Self has derived an analogous formula by assuming that the waist of the input beam represents the object, and the waist of the output beam represents the image. The formula is expressed in terms of the Rayleigh range of the input beam.

In the regular form,

$$\frac{1}{s+z_R^2/(s-f)}+\frac{1}{s''}=\frac{1}{f} \qquad (2.13)$$

or, in dimensionless form,

$$\frac{1}{(s/f)+(z_R/f)^2/(s/f-1)}+\frac{1}{(s''/f)}=1. \qquad (2.14)$$

In the far-field limit as z/R approaches 0 this reduces to the geometric optics equation. A plot of s/f versus s''/f for various values of z/R/f is shown in figure 2.7. For a positive thin lens, the three distinct regions of interest correspond to real object and real image, real object and virtual image, and virtual object and real image.

The main differences between Gaussian beam optics and geometric optics, highlighted in such a plot, can be summarized as follows:

- There is a maximum and a minimum image distance for Gaussian beams.
- The maximum image distance occurs at $s=f+z$/R, rather than at $s=f$.
- There is a common point in the Gaussian beam expression at $s/f=s''/f=1$. For a simple positive lens, this is the point at which the incident beam has a waist at the front focus and the emerging beam has a waist at the rear focus.
- A lens appears to have a shorter focal length as z/R/f increases from zero (i.e., there is a Gaussian focal shift).

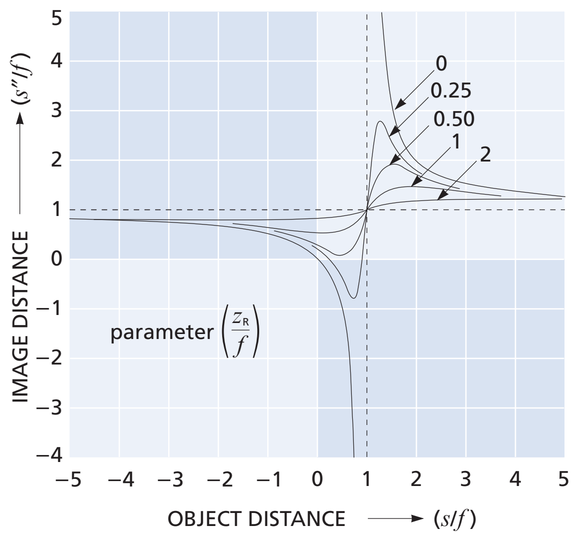

Figure 2.7 **Plot of lens formula for Gaussian beams with normalized Rayleigh range of the input beam as the parameter**

Self recommends calculating z_R, w_0, and the position of w_0 for each optical element in the system in turn so that the overall transformation of the beam can be calculated. To carry this out, it is also necessary to consider magnification: w_0''/w_0. The magnification is given by

$$m=\frac{w_0''}{w_0}=\frac{1}{\sqrt{\left\{\left[1-(s/f)\right]^2+(z_R/f)^2\right\}}}. \qquad (2.15)$$

The Rayleigh range of the output beam is then given by

$$z_R''=m^2 z_R\ . \qquad (2.16)$$

All the above formulas are written in terms of the Rayleigh range of the input beam. Unlike the geometric case, the formulas are not symmetric with respect to input and output beam parameters. For back tracing beams, it is useful to know the Gaussian beam formula in terms of the Rayleigh range of the output beam:

$$\frac{1}{s}+\frac{1}{s''+z_R''^2/(s''-f)}=\frac{1}{f}\ . \qquad (2.17)$$

BEAM CONCENTRATION

The spot size and focal position of a Gaussian beam can be determined from the previous equations. Two cases of particular interest occur when $s = 0$ (the input waist is at the first principal surface of the lens system) and $s = f$ (the input waist is at the front focal point of the optical system). For $s = 0$, we get

$$s'' = \frac{f}{1+\left(\lambda f/\pi w_0^2\right)^2} \tag{2.18}$$

and

$$w = \frac{\lambda f/\pi w_0}{\left[1+\left(\lambda f/\pi w_0^2\right)^2\right]^{1/2}} \tag{2.19}$$

For the case of $s=f$, the equations for image distance and waist size reduce to the following:

$$s'' = f$$

and

$$w = \lambda f/\pi w_0.$$

Substituting typical values into these equations yields nearly identical results, and for most applications, the simpler, second set of equations can be used.

In many applications, a primary aim is to focus the laser to a very small spot, as shown in figure 2.8, by using either a single lens or a combination of several lenses.

If a particularly small spot is desired, there is an advantage to using a well-corrected high-numerical-aperture microscope objective to concentrate the laser beam. The principal advantage of the microscope objective over a simple lens is the diminished level of spherical aberration. Although microscope objectives are often used for this purpose, they are not always designed for use at the infinite conjugate ratio. Suitably optimized lens systems, known as infinite conjugate objectives, are more effective in beam-concentration tasks and can usually be identified by the infinity symbol on the lens barrel.

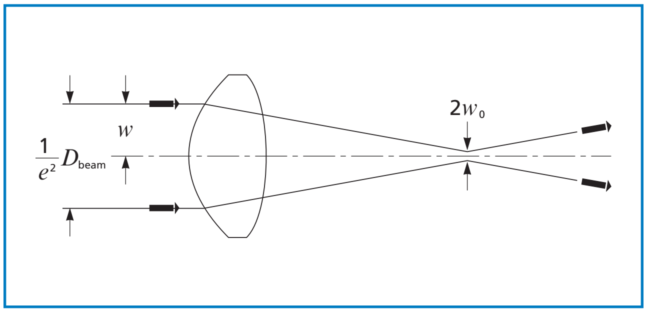

Figure 2.8 **Concentration of a laser beam by a laser-line focusing singlet**

DEPTH OF FOCUS

Depth of focus ($\pm\Delta z$), that is, the range in image space over which the focused spot diameter remains below an arbitrary limit, can be derived from the formula

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}.$$

The first step in performing a depth-of-focus calculation is to set the allowable degree of spot size variation. If we choose a typical value of 5 percent, or $w(z)_0 = 1.05 w_0$, and solve for $z = \Delta z$, the result is

$$\Delta z \approx \pm\frac{0.32\pi w_0^2}{\lambda}.$$

Since the depth of focus is proportional to the square of focal spot size, and focal spot size is directly related to f-number (f/#), the depth of focus is proportional to the square of the f/# of the focusing system.

TRUNCATION

In a diffraction-limited lens, the diameter of the image spot is

$$d = K \times \lambda \times \text{f/\#} \tag{2.20}$$

where K is a constant dependent on truncation ratio and pupil illumination, λ is the wavelength of light, and f/# is the speed of the lens at truncation. The intensity profile of the spot is strongly dependent on the intensity profile of the radiation filling the entrance pupil of the lens. For uniform pupil illumination, the image spot takes on the Airy disc intensity profile shown in figure 2.9.

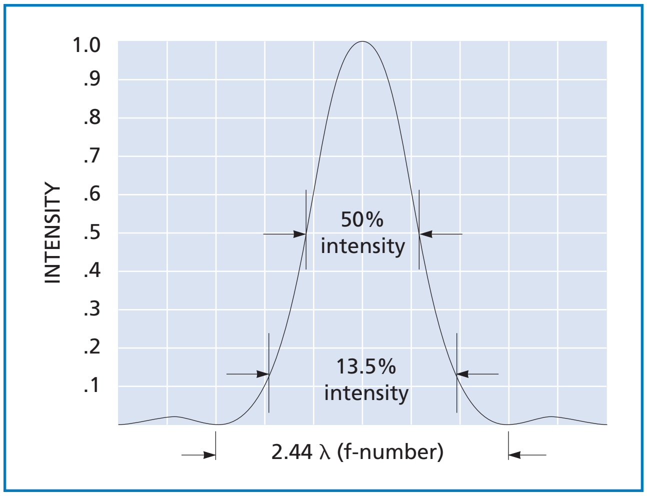

Figure 2.9 **Airy disc intensity distribution at the image plane**

If the pupil illumination is Gaussian in profile, the result is an image spot of Gaussian profile, as shown in figure 2.10.

When the pupil illumination is between these two extremes, a hybrid intensity profile results.

In the case of the Airy disc, the intensity falls to zero at the point $d_{zero} = 2.44 \times \lambda \times f/\#$, defining the diameter of the spot. When the pupil illumination is not uniform, the image spot intensity never falls to zero making it necessary to define the diameter at some other point. This is commonly done for two points:

$d_{FWHM} = $ 50-percent intensity point

and

$d_{1/e^2} = $ 13.5% intensity point.

It is helpful to introduce the truncation ratio

$$T = \frac{D_b}{D_t} \tag{2.21}$$

where D_b is the Gaussian beam diameter measured at the $1/e^2$ intensity point, and D_t is the limiting aperture diameter of the lens. If $T = 2$, which approximates uniform illumination, the image spot intensity profile approaches that of the classic Airy disc. When $T = 1$, the Gaussian profile is truncated at the $1/e^2$ diameter, and the spot profile is clearly a hybrid between an Airy pattern and a Gaussian distribution. When $T = 0.5$, which approximates the case for an untruncated Gaussian input beam, the spot intensity profile approaches a Gaussian distribution.

Calculation of spot diameter for these or other truncation ratios requires that K be evaluated. This is done by using the formulas

$$K_{FWHM} = 1.029 + \frac{0.7125}{(T - 0.2161)^{2.179}} - \frac{0.6445}{(T - 0.2161)^{2.221}} \tag{2.22}$$

and

$$K_{1/e^2} = 1.6449 + \frac{0.6460}{(T - 0.2816)^{1.821}} - \frac{0.5320}{(T - 0.2816)^{1.891}}. \tag{2.23}$$

The K function permits calculation of the on-axis spot diameter for any beam truncation ratio. The graph in figure 2.11 plots the K factor vs $T(D_b/D_t)$.

The optimal choice for truncation ratio depends on the relative importance of spot size, peak spot intensity, and total power in the spot as demonstrated in the table below. The total power loss in the spot can be calculated by using

$$P_L = e^{-2(D_t/D_b)^2} \tag{2.24}$$

for a truncated Gaussian beam. A good compromise between power loss and spot size is often a truncation ratio of $T = 1$. When $T = 2$ (approximately uniform illumination), fractional power loss is 60 percent. When $T = 1$, d_{1/e^2} is just 8.0 percent larger than when $T = 2$, whereas fractional power loss is down to 13.5 percent. Because of this large savings in power with relatively little growth in the spot diameter, truncation ratios of 0.7 to 1.0 are typically used. Ratios as low as 0.5 might be employed when laser power must be conserved. However, this low value often wastes too much of the available clear aperture of the lens.

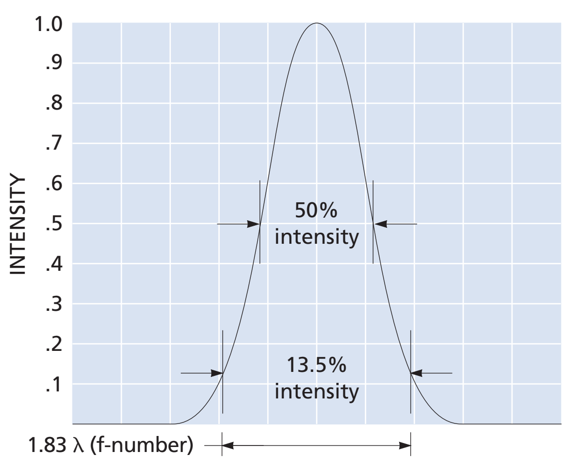

Figure 2.10 **Gaussian intensity distribution at the image plane**

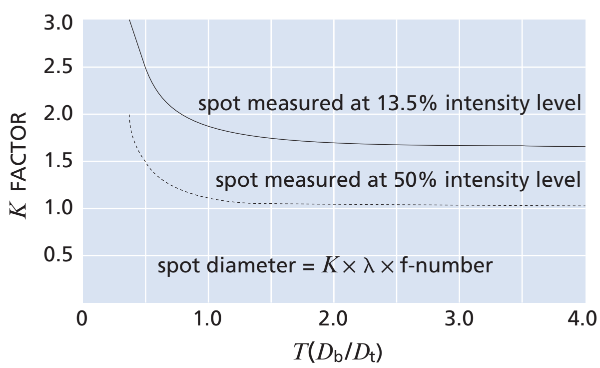

Figure 2.11 ***K* factors as a function of truncation ratio**

Spot Diameters and Fractional Power Loss for Three Values of Truncation

Truncation Ratio	d_{FWHM}	d_{1/e^2}	d_{zero}	P_{L}(%)
Infinity	1.03	1.64	2.44	100
2.0	1.05	1.69	—	60
1.0	1.13	1.83	—	13.5
0.5	1.54	2.51	—	0.03

SPATIAL FILTERING

Laser light scattered from dust particles residing on optical surfaces may produce interference patterns resembling holographic zone planes. Such patterns can cause difficulties in interferometric and holographic applications where they form a highly detailed, contrasting, and confusing background that interferes with desired information. Spatial filtering is a simple way of suppressing this interference and maintaining a very smooth beam irradiance distribution. The scattered light propagates in different directions from the laser light and hence is spatially separated at a lens focal plane. By centering a small aperture around the focal spot of the direct beam, as shown in figure 2.12, it is possible to block scattered light while allowing the direct beam to pass unscathed. The result is a cone of light that has a very smooth irradiance distribution and can be refocused to form a collimated beam that is almost uniformly smooth.

As a compromise between ease of alignment and complete spatial filtering, it is best that the aperture diameter be about two times the $1/e^2$ beam contour at the focus, or about 1.33 times the 99% throughput contour diameter.

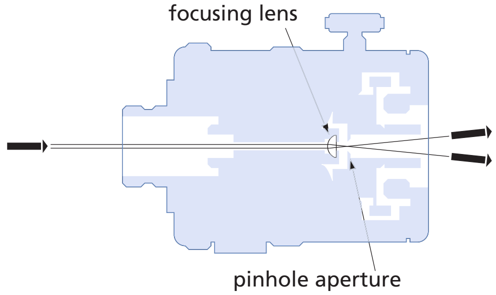

Figure 2.12 **Spatial filtering smoothes the irradiance distribution**

Do you need . . .

SPATIAL FILTERS

CVI Melles Griot offers 3-axis spatial filters with precision micrometers (07 SFM 201 and 07 SFM 701). These devices feature an open-design that provides access to the beam as it passes through the instrument. The spatial filter consists of a precision, differential-micrometer y-z stage, which controls the pinhole location, and a single-axis translation stage for the focusing lens. The spatial filter mount accepts LSL-series focusing optics, OAS-series microscope objectives, and PPM-series mounted pinholes.

The precision individual pinholes are for general-purpose spatial filtering. The high-energy laser precision pinholes are constructed specifically to withstand irradiation from high-energy lasers.