Theory of Tabletop Vibration

The primary goal of optical table design is to eliminate relative motion between any components on the surface. Before the design of optical tables can be discussed, however, it is necessary to examine the underlying theory and to understand the appropriate nomenclature: compliance, vibration, resonance, and damping.

COMPLIANCE

A common problem in engineering and physics is the deformation of a body or structure in response to external forces. These forces may be static or dynamic. The design of an optical tabletop is a good example of this problem. A static force, such as that caused by a large, localized mass placed on the table, can cause the table to sag. A dynamic force, such as the acoustic vibrations in the air, the vibrations of a small motor sitting on top of the table, or vibration induced from the building into the tabletop through its mounting points, can cause the tabletop to vibrate and deform.

Compliance is the most widely used transfer function for the vibrational response of an optical table. In the case of a constant (static) force, this is defined as the ratio of the linear or angular displacement to the magnitude of the applied force. In the case of a dynamically varying force (vibration), the compliance is the ratio of the excited vibrational amplitude (angular or linear displacement) to the magnitude of the forcing vibration. Any deflection of the tabletop is evidenced by the change in relative position of the components mounted on the table surface. Therefore, by definition, lower compliance values mean a better table because the deflection of the surface on which components are mounted is minimized.

Compliance, used to measure deflection at different frequencies, is measured in units of displacement/unit force, i.e., meters/Newton.

Compliance Curves

To understand compliance, consider the hypothetical structure, shown in figure 9.7, with only one vibrational degree of freedom (i.e., a structure with only one direction of deformation). This could be a steel bar, firmly anchored at one end, which can bend only in one plane.

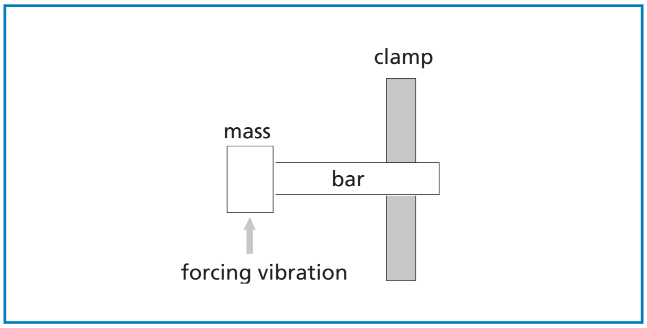

Figure 9.7 **A simple one degree of freedom system: a constrained bar vibrating in only one plane**

All periodic vibrations can be expressed as combinations of sine and cosine functions, together with appropriate amplitude, frequency, and phase. Therefore, consider a single frequency sinusoidal vibration applied to the bar. From Newton's laws, the general equation of motion is

$$ma + cv + kx = F_0 \sin ft \tag{9.1}$$

where the left-hand side pertains to the forced system and the right-hand side pertains to the forcing function, and

a is the acceleration

v is the velocity

x is the displacement

m is the mass being moved or deflected

c is the damping

k is the stiffness

$F_0 \sin ft$ is a sinusoidally varying force with frequency f, maximum amplitude F_0, and time t.

The general expression for compliance of a system such as this is given by

$$\text{compliance} = \frac{x}{F} = \frac{1}{\left[\left(k - mf^2\right) + \left(cf\right)^2\right]^{1/2}} . \tag{9.2}$$

If we restate the above equation in words,

$$\text{compliance} = \frac{1}{\sqrt{(\text{stiffness} - \text{mass effects}) + \text{damping}}} . \tag{9.3}$$

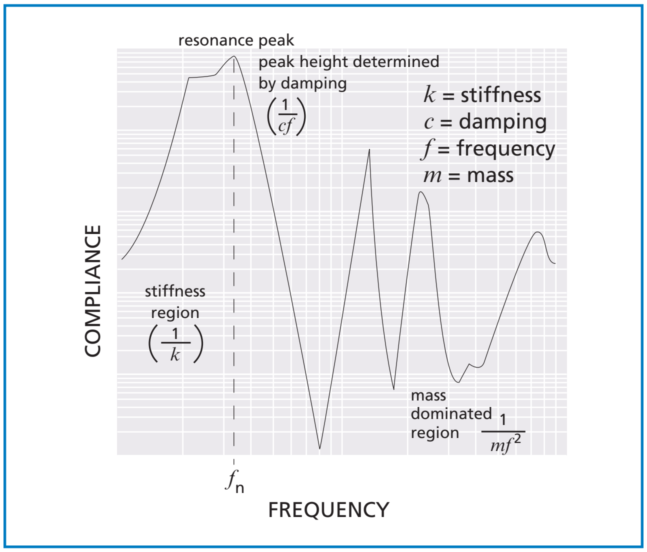

Figure 9.8 **Compliance versus frequency for a one degree of freedom system**

A plot of compliance versus frequency shows that the compliance of a rigid body can be separated into three parts: stiffness, resonance effects, and mass effects, as shown in figure 9.8.

Compliance at Low Frequencies—Stiffness

At low frequencies, the stiffness term dominates the compliance equation. When a low-frequency forcing vibration is applied to the unattached end of our bar, it bends in response. The amount of deflection is determined by the stiffness of the bar which ultimately depends on its shape, the tensile modulus of elasticity (Young's modulus) of the bar material, and the method of mounting and/or constraining the bar.

Any solid body has a fixed equilibrium (rest) geometry which is what distinguishes a true solid from a liquid. The equilibrium geometry is that which minimizes the potential energy. When forces are applied to a solid body it can be deformed from this equilibrium shape; however, the potential energy of the body rises and is manifested as resistive forces, which act to restore the equilibrium geometry.

When the bar is deflected, the restoring forces attempt to return it to its equilibrium position; however, the momentum of the bar causes it to overshoot the equilibrium position. Restoring forces then act in the opposite direction to return the bar to its equilibrium position. Again, momentum causes the bar to overshoot. This oscillation is an example of a simple harmonic oscillator. The oscillation occurs with a characteristic frequency f_n—the resonant frequency—given by

$$f_n = \sqrt{\frac{k}{m}}\ . \qquad (9.4)$$

In the absence of damping, this oscillation would persist forever. When a body is vibrating at its resonant frequency, the energy in the vibration alternates between potential and kinetic. At the maximum deflection or displacement, the velocity is zero, the acceleration is maximum, the potential energy is maximum, and the kinetic energy is zero. At the equilibrium, or zero displacement, the opposite is true.

In a real system, such as an optical table, resonant vibrations rarely approximate harmonic vibrations with such simple mathematics; however, the discussion above is still valid.

Compliance at Resonance

When the forcing vibration is at the resonant frequency, each maximum in the velocity of the forcing vibration coincides with a maximum in the acceleration of the excited vibration. This adds to the acceleration of the bar, which thereby accumulates vibrational energy and actually amplifies the forcing vibration. This is in accordance with equation 9.2, which can be solved to show

$$\text{compliance} = \frac{1/k}{\left[\left(1 - f^2/f_n^2\right)^2 + \left(2\,\zeta f/f_n\right)^2\right]^{1/2}} \qquad (9.5)$$

where f is the frequency of the forcing function and ζ is the damping ratio. From this equation, it can be seen that when the frequency of the forcing function is close to the resonant frequency, the compliance is determined solely by the damping term and can be quite large.

Another way to picture resonance is to consider what happens as the forcing function is raised slowly from zero, as shown in figure 9.9. At forcing frequencies near zero, the bar bends synchronously with the forcing vibration. As the frequency is increased, the bar starts to lag behind the forcing vibration because it has momentum and cannot reverse direction instantaneously in response to the periodic direction changes in the applied force (i.e., vibration). The bar eventually halts and reverses direction due to its own restoring forces. The same thing happens at the opposite extreme of the motion.

The phase lag continues to increase with frequency until it is exactly 90 degrees at the resonant frequency. At this point, the balance of the bar momentum and its restoring forces causes the maximum in the bar acceleration to coincide with (and be in the same direction as) the maximum in the velocity of the forcing vibration, resulting in amplification of the vibrational input.

At forcing vibration frequencies greater than the resonant frequency, the phase difference between forcing and excited vibrations in this theoretical single-resonance system is 180 degrees.

In a real structure such as an optical table, there are many possible resonant modes of vibration. The phase of an excited vibration is also dependent upon where on the table it is measured, and upon what type of vibration is being excited.

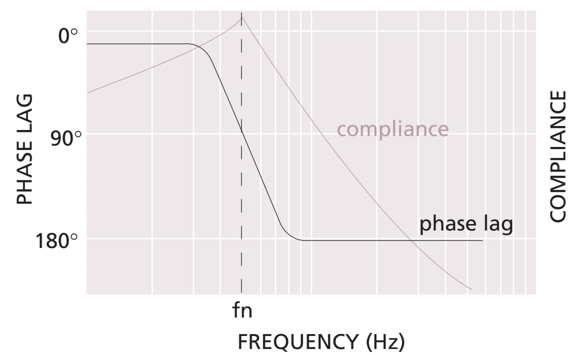

Figure 9.9 **The variation of phase lag between excited and forcing vibrations for a system with a single resonance**

MODIFYING A RESONANT FREQUENCY

Ignoring damping for the moment, the compliance curve is dominated by stiffness at low frequencies and by mass effects above the resonant frequency. The frequency of resonance can be determined from the difference between these two terms in the equation for compliance, and as we saw in equation 9.4, is given by

$$f_n = \sqrt{\frac{k}{m}} \; .$$

The quantity (k/m) is known as the stiffness-to-mass ratio. The resonant frequency of a simple system with one degree of freedom can be altered by changing this stiffness-to-mass ratio. Decreasing the mass and/or increasing the stiffness shifts the resonance to a higher frequency and vice versa, as shown in figure 9.10.

The balance of the mass and stiffness is a simple, yet important, concept in optical table design because the goal is to push the table resonances to higher frequency and lower amplitude. Clearly, increasing the stiffness of the table by proportionally increasing the mass will not shift the resonance. The designer's goal is to maximize the stiffness while minimizing the mass.

DAMPING

Damping refers to any process that causes an oscillation in a solid body to decay to zero amplitude. It is a very important phenomenon in vibration suppression or isolation in real systems because it causes energy to be diverted from vibration to other energy sinks.

Damping is a resonant effect, inasmuch as it significantly affects the compliance function at or near resonance, i.e, when $f \cong f_n$. The height of the compliance peak at resonance is determined primarily by the amount of damping. In the absence of any damping, the peak would be infinitely high, and the system would vibrate ad infinitum; however, the natural damping of the materials themselves prevents this.

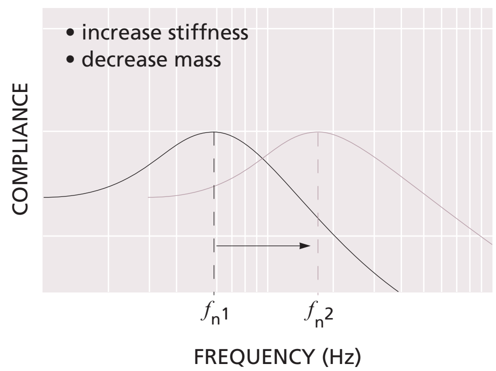

Figure 9.10 **Shifting the resonant frequency by changing the stiffness to mass ratio**

The microstructure of a well-damped material is such that deformations cause strains in the material, which rapidly degrade the energy as heat (i.e., internal energy). Materials such as wood and rubber have a large amount of natural damping. Metals manifest a small amount of internal damping caused by a small amount of friction at grain boundaries.

In many instances a structure may consist of supporting elements which are quite rigid, with little natural damping, and tend to ring. If the structure is being designed to avoid vibrations, additional damping is required. There are many methods of introducing damping into a system, and typically they rely on using friction to degrade the vibrational energy into heat.

Real structures have multiple resonant modes. Usually these modes are coupled, making the form of the resonant vibration quite complex, particularly in the case of nonsymmetrical structures. In addition, each vibrational mode often produces a whole series of harmonic vibrations. Nonetheless, the theory described in the preceding notes is still valid. To summarize:

- At low frequencies, below the first resonant frequency, the compliance of a real structure is determined by the stiffness. Stiffer structures are less compliant (i.e., they vibrate less for a given forcing vibration).
- At frequencies above the region of the first resonance, the compliance of a rigid structure is given roughly by $1/mf^2$ although resonant effects are superimposed upon this.
- The compliance at the various resonant peaks is dependent mainly on the amount of damping at these resonant frequencies and by the effective mass associated with the vibrational mode. However, the relative height of these peaks is still approximately proportional to $1/f^2$.

At higher frequencies, the compliance is totally dominated by the mass (inertia) of the structure. From equation 9.2, the compliance at high frequencies becomes

$$\text{compliance} = \frac{1}{mf^2} \; . \tag{9.6}$$

Although in certain very simple vibrational systems (e.g., a single mass suspended from a spring) it may be fairly easy to evaluate the mass involved in the vibration, in most real-world cases, determining the mass can be very complex and is best determined by computer programs. For example, an optical table has several types of bending and flexural resonant modes of vibration, and different points on the table have different vibrational amplitudes. Furthermore, at nodal points, there is no vibrational amplitude at all.

ANTIRESONANCES

In systems, such as an optical table, with several vibrational degrees of freedom, the compliance curve will often show negative peaks between the resonant peaks. These are usually overlooked in most discussions of optical tables, yet they give an insight into the concept of compliance phase.

The phase of the vibrational response of the table changes from 0 to 180 degrees as the compliance curve passes through a resonance. The *stiffness* part of the curve is in phase with the forcing function and the *mass* part of the curve is exactly out of phase with the forcing function. Now consider the case of a system with two resonances (of similar compliance) at different frequencies as shown in figure 9.11. The mass part of one resonance occurs at the same frequency as the stiffness part of the other resonance; therefore, it is possible for the net compliance at this point to be very small. The negative peak, termed an *antiresonance*, and may almost reach zero if the two terms are very similar in magnitude.

COMPLIANCE OF A REAL TABLE

The concept of an ideal rigid body is useful when considering optical table performance. This theoretical structure does not resonate and therefore has no compliance peaks. When plotted on a log:log scale, an ideal rigid body has a compliance proportional to $1/f^2$ and is represented by a straight line with a slope of -2. It represents the ultimate design goal when manufacturing optical tables—the nearer the actual curve fits the straight line the better the dynamic stiffness.

A compliance curve of a real table is shown in figure 9.12. The measurements were taken at the corner of the tabletop. Conventionally, this type of data is presented on a double logarithmic scale—vertically because of the large range of compliance values and horizontally in order to display a wide range of frequencies while highlighting the important region around the low-frequency resonances.

Several aspects of this curve merit special comment. The initial portion of the plot, before the first table resonance, is determined primarily by the

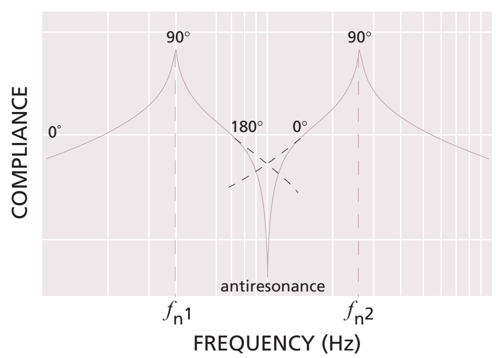

Figure 9.11 **An antiresonance, caused by two vibrational components of equal magnitude that are 180 degrees out of phase**

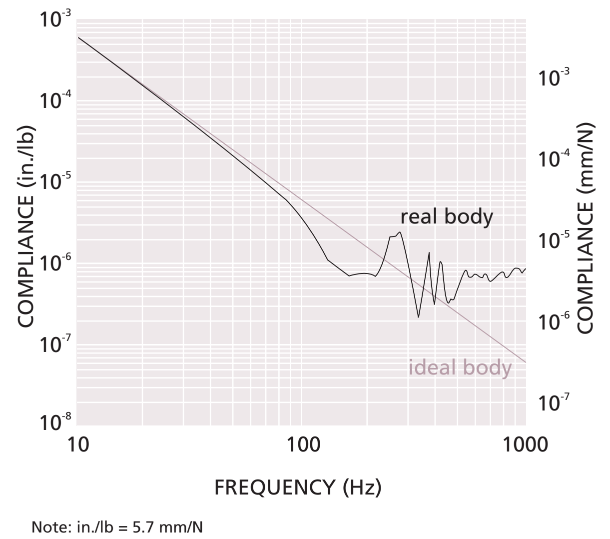

Figure 9.12 **A typical compliance curve of a high-performance CVI Melles Griot optical table**

table supports, not by the table itself. Notice how the peaks at compliance resonances decrease in size toward higher frequency. As the frequency increases, the denominator in the compliance expression (see equation 9.2) becomes large and therefore the compliance is reduced. This means that, as the frequency increases, a given excitation force produces a smaller amplitude excitation in the table. The higher-frequency vibrations of a table are the various combinations and overtones of the fundamental vibrations of the table, discussed below.

The low-frequency peaks are the most important because they are the largest peaks, corresponding to the weakest points in the compliance spectrum; and typical vibrations from laboratory equipment are usually below 150 Hz. These peaks should be at the highest frequency possible in order to keep the compliance in the 0 to 150 Hz region as low as possible.

In this particular curve, notice the inverted peak at around 120 Hz. This is a typical antiresonance caused by the phase effects described earlier.

It should be noted also that a single compliance curve does not describe the performance of an optical table. A few modes may have very low, or zero, amplitudes at the corner of measurement. In general, however, a compliance curve obtained at the corner of an optical table is a reasonable measure of the worst-case performance of any part of the table surface.

It is always difficult to specify a piece of equipment by a convoluted curve or spectrum as opposed to a number or series of numbers. Consequently, two values are used to specify the performance of an optical table: the frequency of the first resonance, which should be as high a value as possible, and the peak compliance, the height of the highest (usually the first) peak.

TABLE STIFFNESS

As previously discussed, the measured low-frequency compliance of an optical table is entirely a function of the table support structure used during the test. The low-frequency behavior of an optical table is usually expressed as the stiffness or deflection under a measured static load. This can be measured as either a displacement of the center of the table or as an angle of deformation of the tabletop. The actual values depend on the relative position of the supports.

TABLE THICKNESS

Compliance also varies with table thickness. The graph in figure 9.13 shows that, for a 1.5 × 3-m table, if the thickness is increased from 210 to 420 mm, the compliance improves from 6×10^{-5} mm/N to 1.4×10^{-5} mm/N.

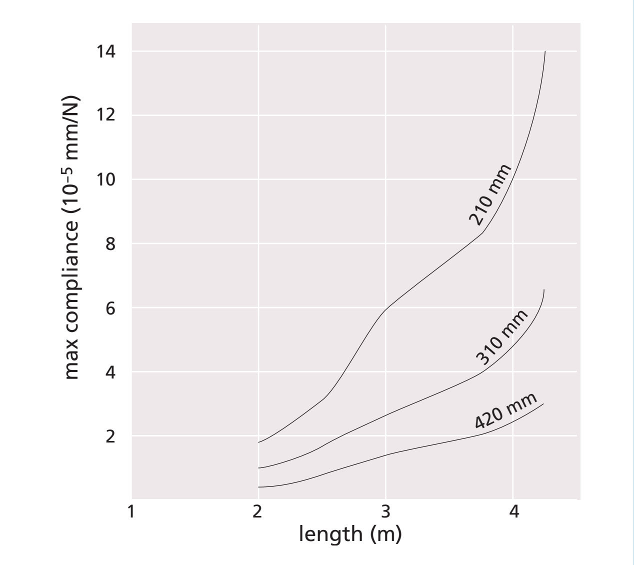

Figure 9.13 **Compliance vs length for different thickness tabletops**

Table compliance improves with thickness

Many applications require a high degree of stability