Theory of Vibration Isolation

Vibration isolation or elimination on an optical surface is a two-part problem. As discussed previously, an optical tabletop is designed to have zero or minimal response to a deflective force or vibration. This in itself is not sufficient to ensure a vibration-free working surface. The rigid tabletop may still vibrate without deforming (i.e., vibrations of the table on the mounting system). These vibrations are constrained translations and/or rotations of the optical table.

Typically, the entire table system is subjected continually to vibrational impulses from the laboratory floor. These vibrations may be caused by large machinery within the building or even by wind or traffic-excited building resonances (swaying).

SEISMIC MOUNTING

Vibrations of a floor in a building can be divided into two basic components: vertical and horizontal. Typically, vertical components range from 10 to 50 Hz, and horizontal components range from 1 to 20 Hz. To prevent such vibrations from disturbing an experiment, it is important to mount the table in such a way as to isolate it vibrationally from the laboratory floor (i.e., mount the table in such a way that its instantaneous position is independent of the periodic motions of the laboratory floor). This type of isolation is termed "seismic" mounting. When an object is truly seismically mounted with respect to the floor, the motions of the object and the floor are completely uncoupled and separate. The term seismic is linked with the study of earthquakes, in which the magnitude of an earthquake is estimated from the motion of the ground with respect to a seismically mounted indicator.

TRANSMISSIBILITY

The simplest model used in a theoretical treatment of mechanical vibration is the ball and spring as shown in figure 9.24. Consider a ball suspended from an enormous mass by a spring. For now, we shall ignore pendulum motions and consider a system having only one degree of freedom—capable only of vertical extension.

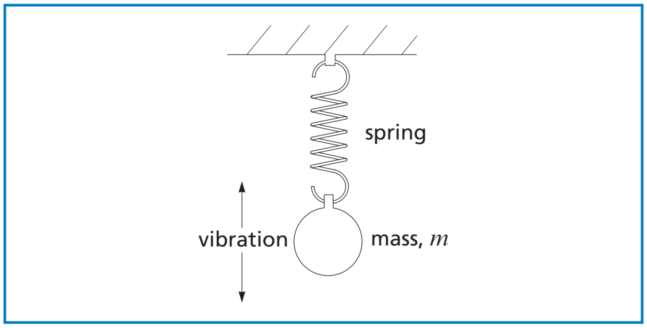

Figure 9.24 **A ball and spring example of a vibration isolation/transmission system with one degree of freedom**

In the absence of vibrational impulses, the ball is stationary at its rest position. Suppose the object from which the ball is suspended is not infinitely large or not infinitely stiff so that the point at which the spring is anchored starts to vibrate. Some of the vibration energy may be felt at the ball, causing it to vibrate at the same frequency. The frequency of this motion is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (9.7)$$

where f_n is the resonant frequency of the oscillation, m is the mass moving during the oscillation, and k is the spring constant (related to the stiffness of the spring).

The flow of vibrational energy is expressed in terms of a transfer function. A transfer function is a method of quantifying how efficiently a forcing vibration can produce an excited vibration. The transfer function most applicable here, transmissibility, is defined as the ratio of the dynamic output to the dynamic input, i.e., the ratio of the amplitude of the transmitted vibration to that of the forcing vibration.

RESONANCE

In the example just outlined, the transmissibility of the spring is very dependent on the frequency of the forcing vibration. The idealized transmissiblity of this system is given by

$$T = \sqrt{\frac{1+(2\zeta f/f_n)^2}{(1-f^2/f_n^2)^2+(2\zeta f/f_n)^2}} \qquad (9.8)$$

where: f is the frequency of the forcing function, f_n is the natural resonant frequency of the system, and ζ is the damping ratio.

The graph shown in figure 9.25 is an idealized plot of transmissibility as a function of frequency. Note the similarity to the compliance transfer function previously discussed in the table vibration text. Again there are three distinct regions on the curve.

At low or zero frequencies, the ball and spring in the previous example move synchronously with the table and with the same amplitude; i.e., the transmissibility is unity as defined by the preceding equation. The system behaves as though the spring were rigid and there were no vibrational isolation. As the frequency increases a resonant condition is approached. The ball has mass and momentum when moving and cannot change direction instantaneously in response to a force whose direction is rapidly changing, i.e., the forcing vibration. The vibration of the ball starts to lag behind the forcing vibration, i.e., they are no longer in phase. Eventually, this phase lag becomes exactly 90 degrees and at this point, the system is vibrating at its natural or resonant frequency, as shown in figure 9.26.

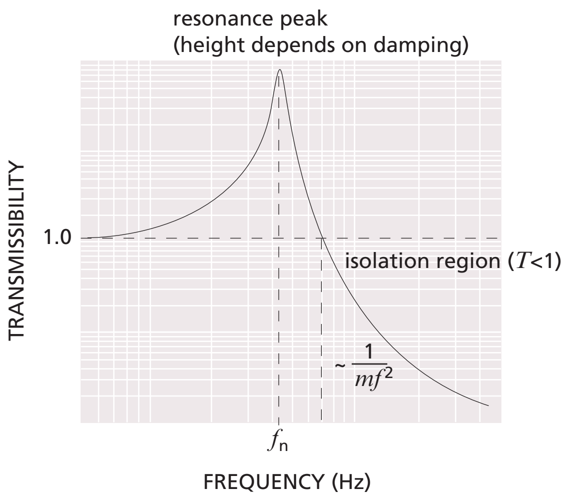

Figure 9.25 **A typical transmissibility vs frequency curve for a system with one degree of freedom**

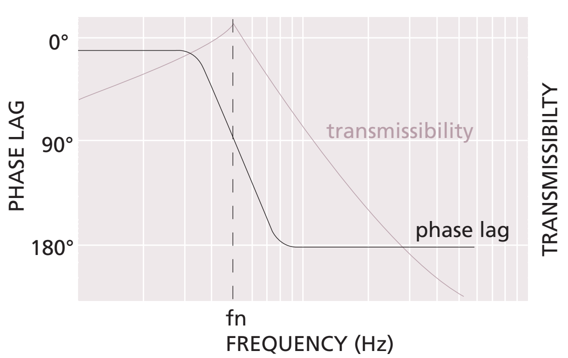

Figure 9.26 **The phase relationship between excited and forcing vibrations changes rapidly near resonance.**

An example of a structure at natural or resonant frequency is a simple tuning fork. It is the frequency at which the excited momentum and kinetic energy of the motion are in perfect dynamic equilibrium with the restoring force (potential energy). The system oscillates between high kinetic energy-zero potential energy and high potential energy-zero kinetic energy. At its resonant frequency, the system accumulates vibrational energy and increases in amplitude during the time the forcing vibration is applied. At resonance, the system acts as a vibrational amplifier because of the 90-degree phase lag: the maximum velocity of the forcing vibration coincides with maximum acceleration of the excited vibration. As defined by the equation for transmissibility, the amplitude of this resonant motion does not grow to infinity because of damping.

At frequencies much higher than the resonant frequency, the response of the ball is determined solely by the mass term, which is much larger than the stiffness term. In other words, the spring is relatively soft and the vibrational force travels slowly along it in the form of compression/extension waves. This slow transmission effectively spreads out the oscillatory nature of the forcing vibration. Essentially, the ball experiences a time-averaged force owing to the fast moving vibration and, unless the vibration involves a net displacement, this time-averaged force tends to zero with increasing vibrational frequency. As the transmissibility approaches zero, the position of the ball is not affected by the vibration in the large mass. This is a classic example of seismic mounting.

DAMPING

Damping refers to any process that causes an oscillation in a system to decay rapidly to zero amplitude. It is a very important phenomenon in vibration suppression or isolation. Damping causes the energy to be diverted from vibration to other energy sinks. The damping ratio of a system (ζ) is defined as the ratio of actual damping (C) to critical damping (C_c). Critical damping is the minimum amount of damping in a system necessary to prevent resonant oscillation following application of an impulsive force.

Damping is a resonant effect in that, primarily, it affects the transmissibility function at or near resonance.

At f/f_n=1, resonance, the transmissibility becomes

$$T = \sqrt{\frac{1}{(2\zeta)^2} + 1} \qquad (9.9)$$

where ζ is the damping ratio, (C/C_c).

For a lightly damped system, i.e., $\zeta < 0.1$, the maximum transmissibility is

$$T = \frac{1}{2\zeta} \; . \qquad (9.10)$$

The height of the transmissibility peak at resonance is determined primarily by the amount of damping. In the absence of damping, the peak would be infinitely high. Also, a system with no damping would vibrate resonantly ad infinitum, even when the vibrational source (forcing function) is removed. Clearly, all real systems contain damping to some degree. Using the suspended ball as an example, the simplest form of damping would be to immerse the ball in a viscous oil. Viscous drag would convert the vibrational energy to heat by friction.

SEISMIC MOUNTING OF OPTICAL TABLES

The preceding discussion suggests in principle, a way in which an optical table could be vibrationally isolated (seismically mounted). If an optical tabletop could be suspended from the ceiling by springs, then at frequencies much higher than the natural resonance of the spring isolation system, ambient building vibrations would not be transmitted to the tabletop. Obviously, this solution has no practical value, but it clarifies the general principles involved.

Typically, ambient vibrations in a building tend to be in the range from 10 to 50 Hz for vertical components and from 1 to 20 Hz for horizontal components. To mount an optical table seismically under these conditions requires a mounting system with a very low resonant frequency (i.e., a spring with a small spring constant). Unfortunately, even with the use of composites, optical tables are relatively massive—typically 500 kg for a 1.2 × 2.4-m table. Such a system would involve highly extended springs and a very large travel range. However, it is interesting to note that if stiff springs were used in such a system, then the resonant frequency for pendulum motion could easily be less than 1 Hz; i.e., the tabletop would be well isolated from horizontal vibrations of the building.

The ideal mounting system, shown in figure 9.27, has a rigid tabletop mounted to the floor in such a way that it can vibrate in vertical and horizontal directions, both with low resonant frequencies. Even though these resonant frequencies can be made lower than most building vibrations, it is important that the height, and hence the width, of the resonant peak in the transmissibility curve be reduced. This is achieved by adding a damping mechanism which ensures minimum oscillation at resonance and short settling times.

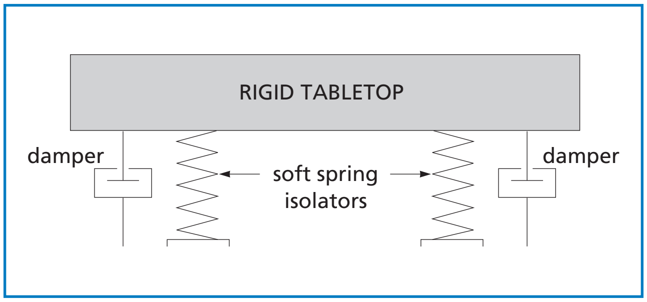

Figure 9.27 **Ideal seismic mounting of an optical table, consisting of weak spring supports with added damping**

Rigid, passive and self leveling table supports

Low noise, low vibration compressor can be used when compressed air is not available

Fundamentals of Vibration Isolation