

Waveplates

Waveplates use birefringence to impart unequal phase shifts to the orthogonally polarized field components of an incident wave, causing the conversion of one polarization state into another.

There are two types of birefringence. With linear birefringence, the index of refraction (and hence the phase shift) differs for two orthogonally polarized linear polarization states. This is the operation mode of standard waveplates. With circular birefringence, the index of refraction and hence phase shift differs for left and right circularly polarized components. This is the operation mode of polarization rotators.

STANDARD WAVEPLATES: LINEAR BIREFRINGENCE

Suppose a waveplate made from a uniaxial material has light propagating perpendicular to the optic axis. This makes the field component parallel to the optic axis an extraordinary wave and the component perpendicular to the optic axis an ordinary wave. If the crystal is positive uniaxial, $\eta_e > \eta_o$, then the optic axis is called the slow axis, which is the case for crystal quartz. For negative uniaxial crystals $\eta_e < \eta_o$, the optic axis is called the fast axis.

The equation for the transmitted field E_2 , in terms of the incident field E_1 is:

$$E_2 = s(s \cdot E_1)e^{i\phi_s} + f(f \cdot E_1)e^{i\phi_f} \quad (1.102)$$

where s and f are unit vectors along the slow and fast axes. This equation shows explicitly how the waveplate acts on the field. Reading from left to right, the waveplate takes the component of the input field along its slow axis and appends the slow axis phase shift to it. It does a similar operation to the fast component.

The slow and fast axis phase shifts are given by:

$$\phi_s = \eta_s(\omega)\omega t / c = 2\pi\eta_s(\lambda)t / \lambda \quad (1.103)$$

$$\phi_f = \eta_f(\omega)\omega t / c = 2\pi\eta_f(\lambda)t / \lambda \quad (1.104)$$

where η_s and η_f are, respectively, the indices of refraction along the slow and fast axes, and t is the thickness of the waveplate.

To further analyze the effect of a waveplate, we throw away a phase factor lost in measuring intensity, and assign the entire phase delay to the slow axis:

$$E_2 = s(s \cdot E_1)e^{i\phi} + f(f \cdot E_1) \quad (1.105)$$

$$\phi = \phi_s - \phi_f = 2\pi[\eta_s(\lambda) - \eta_f(\lambda)]t / \lambda \quad (1.106)$$

$$\phi = 2\pi\Delta\eta(\lambda)t / \lambda \quad (1.107)$$

In the above, $\Delta\eta(\lambda)$ is the birefringence $\eta_s(\lambda) - \eta_f(\lambda)$. The dispersion of the birefringence is very important in waveplate design; a quarter waveplate at a given wavelength is never exactly a half waveplate at half that wavelength.

Let E_1 be initially polarized along x , and let the waveplate slow axis make an angle θ with the x axis. This orientation is shown in Figure 1.52.

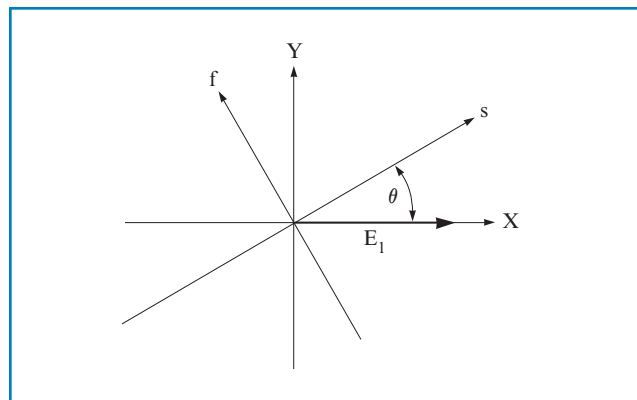


Figure 1.52. Orientation of the slow and fast axes of a waveplate with respect to an x -polarized input field

When the waveplate is placed between parallel and perpendicular polarizers the transmissions are given by:

$$T_{\parallel} \propto |E_{2x}|^2 = 1 - \sin^2 2\theta \sin^2 \phi / 2 \quad (1.108)$$

$$T_{\perp} \propto |E_{2y}|^2 = \sin^2 2\theta \sin^2 \phi / 2 \quad (1.109)$$

Note that θ is only a function of the waveplate orientation, and ϕ is only a function of the wavelength, the birefringence is a function of wavelength and the plate thickness.

For a full-wave waveplate:

$$\phi = 2m\pi, T_{\parallel} = 1, \text{ and } T_{\perp} = 0, \text{ regardless of waveplate orientation.}$$

For a half-wave waveplate:

$$\phi = (2m + 1)\pi, T_{\parallel} = \cos^2 2\theta, \text{ and } T_{\perp} = \sin^2 2\theta. \quad (1.110)$$

This transmission result is the same as if an initial linearly polarized wave were rotated through an angle of 2θ . Thus, a half-wave waveplate finds use as a polarization rotator.

For a quarter waveplate,

$$\phi = (2m + 1)\pi/2 \text{ (i.e., an odd multiple of } \pi/2\text{)}. \quad (1.111)$$

To analyze this, we have to go back to the field equation. Assume that the slow and fast axis unit vectors s and f form a right handed coordinate system such that $s \times f = +z$, the direction of propagation. To obtain circularly polarized light, linearly polarized light must be aligned midway between the slow and fast axes. There are four possibilities listed in the table below.

Phase Shift	Input Field Along $(s + f)/\sqrt{2}$	Input Field Along $(s - f)/\sqrt{2}$
$\phi = \pi/2 + 2m\pi$	RCP	LCP
$\phi = 3\pi/2 + 2m\pi$	LCP	RCP

Sometimes, waveplates described by the second line above are called $3/4$ waveplates. For multiple order waveplates, CVI Melles Griot permits the use of either of the above classes of waveplates to satisfy the requirements of a quarter-wave waveplate.

MULTIPLE-ORDER WAVEPLATES

For the full-, half-, and quarter-wave waveplate examples given in standard waveplates, the order of the waveplate is given by the integer m . For $m > 0$, the waveplate is termed a multiple-order waveplate. For $m = 0$, we have a zero order waveplate.

The birefringence of crystal quartz near 500 nm is approximately 0.00925. Consider a 0.5-mm-thick crystal quartz waveplate. A simple calculation shows that this is useful as a quarter waveplate for 500 nm; in fact, it is a $37 \lambda/4$ waveplate at 500 nm with $m = 18$. Multiple-order waveplates are inexpensive, high-damage-threshold retarders. Further analysis shows that this same 0.5mm plate is a $19 \lambda/2$ half waveplate at 488.2 nm and a 10λ full-wave waveplate at 466.5 nm. The transmission of this plate between parallel polarizers is shown in Figure 1.53 as a function of wavelength. The retardance of the plate at various key points is also shown. Note how quickly the retardance changes with wavelength. Because of this, multiple-order waveplates are generally useful only at their design wavelength.

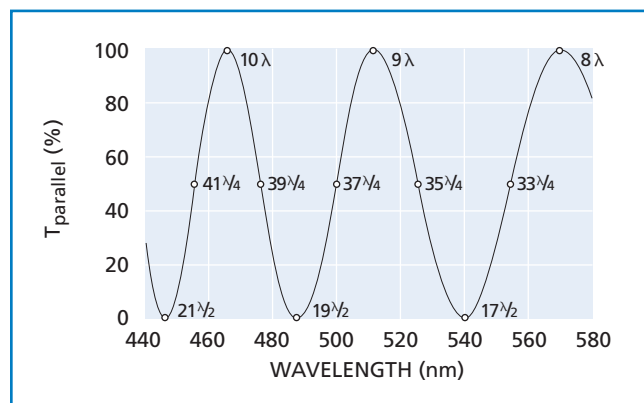


Figure 1.53 **Transmission of a 0.5-mm-thick crystal quartz waveplate between parallel polarizers**

ZERO-ORDER WAVEPLATES

As discussed above, multiple-order waveplates are not useful with tunable or broad bandwidth sources (e.g., femtosecond lasers). A zero-order waveplate can greatly improve the useful bandwidth in a compact, high-damage-threshold device.

As an example, consider the design of a broadband half-wave waveplate centered at 800 nm. Maximum tuning range is obtained if the plate has a single π phase shift at 800 nm. If made from a single plate of crystal quartz, the waveplate would be about $45 \mu\text{m}$ thick, which is too thin for easy fabrication and handling. The solution is to take two crystal quartz plates differing in thickness by $45 \mu\text{m}$ and align them with the slow axis

of one against the fast axis of the other. The net phase shift of this zero-order waveplate is π . The two plates may be either air-spaced or optically contacted. The transmission of an 800-nm zero-order half-wave waveplate between parallel polarizers is shown in Figure 1.54 using a 0-10% scale. Its extinction is better than 100:1 over a bandwidth of about 95 nm centered at 800 nm.

CVI Melles Griot produces multiple order and zero order crystal quartz waveplates at any wavelength between 193 nm and 2100 nm. Virtually all popular laser wavelengths are kept in stock, and custom wavelength parts are available with short delivery time.

Mica waveplates are an inexpensive zero-order waveplate solution for low-power applications and in detection schemes.

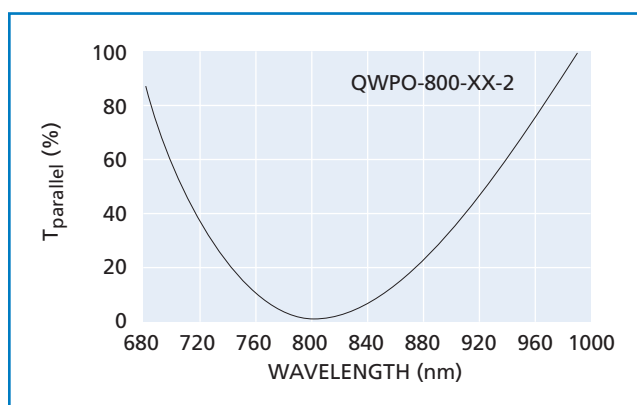


Figure 1.54 **Zero-order crystal quartz half-wave waveplate for 800 nm**

POLYMER WAVEPLATES

Polymer waveplates offer excellent angular field of view since they are true zero-order waveplates. Figure 1.55 compares the change in retardance as function of incidence angle for polymer and quartz waveplates. A polymer waveplate changes by less than 1% over a $\pm 10^\circ$ incidence angle.

Retardance accuracy with wavelength change is often of key concern. For example, an off-the-shelf diode laser has a center wavelength tolerance of ± 10 nm. Changes with temperature and drive conditions cause wavelength shifts which may alter performance. These polymer waveplates maintain excellent waveplate performance even with minor shifts in the source wavelength. The temperature sensitivity of laminated polymer waveplates is about $0.15 \text{ nm}/^\circ\text{C}$, allowing operation over moderate temperature ranges without significantly degrading retardance accuracy. A comparison of different waveplate types and their dependence on wavelength is shown in figure 1.56.

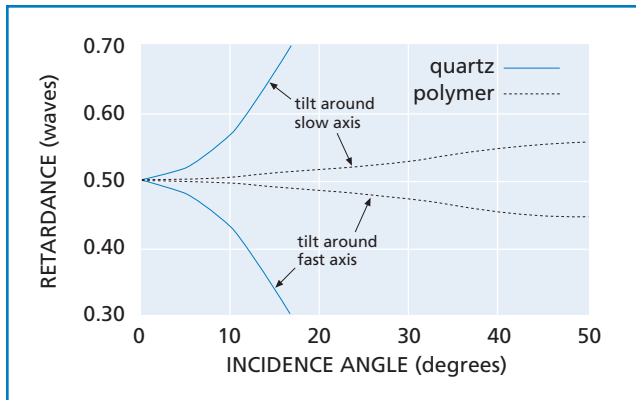


Figure 1.55 Retardance vs incidence angle for quartz and polymer waveplates

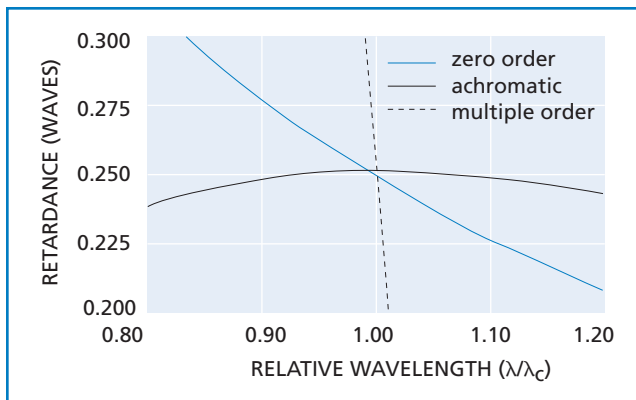


Figure 1.56 Wavelength performance of common quarter wave retarders.

ACHROMATIC WAVEPLATES

At 500 nm, a crystal quartz zero-order half-wave waveplate has a retardation tolerance of $\lambda/50$ over a bandwidth of about 50 nm. This increases to about 100 nm at a center wavelength of 800 nm. A different design which corrects for dispersion differences over the wavelength range is required for bandwidths up to 300 nm.

If two different materials are used to create a zero-order or low-order waveplate, cancellation can occur between the dispersions of the two materials. Thus, the net birefringent phase shift can be held constant over a much wider range than in waveplates made from one material. ACWP-series achromatic waveplates from CVI Melles Griot (see Figure 1.57) are comprised of crystal quartz and magnesium fluoride to achieve achromatic performance.

Three wavelength ranges are available in both quarter and half wave retardances. Retardation tolerance is better than $\lambda/100$ over the entire wavelength range.

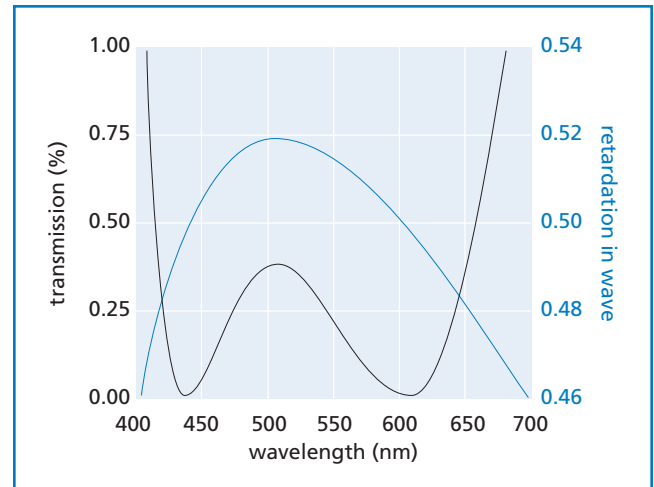


Figure 1.57 ACWP-400-700-10-2

For quarter-wave waveplates, perfect retardance is a multiple of 0.25 waves, and transmission through a linear polarizer must be between 33% and 67%. (In all but the shortest wavelength design, quarter-wave retardation tolerance is better than $\lambda/100$.) For half-wave waveplates, perfect retardance is 0.5 waves, while perfect transmission through a linear polarizer parallel to the initial polarization state should be zero. A high degree of achromatization is achievable by the dual material design. In addition, we manage low group velocity dispersion for ultrashort pulse applications through the use of thin plates.

DUAL-WAVELENGTH WAVEPLATES

Dual-wavelength waveplates are used in a number of applications. One common application is separation of different wavelengths with a polarizing beamsplitter by rotating the polarization of one wavelength by 90° , and leaving the other unchanged. This frequently occurs in nonlinear doubling or tripling laser sources such as Nd:YAG (1064/532/355/266).

One way to achieve the multiple retardation specifications is through careful selection of multiple-order waveplates which meet both wavelength and retardation conditions. This often results in the selection of a relatively high order waveplate. Therefore, these dual-wavelength waveplates operate best over a narrow bandwidth and temperature range.

Another approach is to combine two quartz waveplates with their optical axes orthogonal to one another, effectively creating a zero-order waveplate. In this configuration, the temperature dependence is a function of the thickness difference between the waveplates, resulting in excellent temperature stability. The retardation of the compound waveplate is also a function of the thickness difference enabling wide bandwidth performance.