Real Beam Propagation

In the real world, truly Gaussian laser beams are very hard to find. Low-power beams from helium neon lasers can be a close approximation, but the higher the power of the laser is, the more complex the excitation mechanism (e.g., transverse discharges, flash-lamp pumping), and the higher the order of the mode is, the more the beam deviates from the ideal.

To address the issue of non-Gaussian beams, a beam quality factor, M^2, has come into general use.

For a typical helium neon laser operating in TEM_{00} mode, M^2 <1.1. Ion lasers typically have an M^2 factor ranging from 1.1 to 1.7. For high-energy multimode lasers, the M^2 factor can be as high as 10 or more. In all cases, the M^2 factor affects the characteristics of a laser beam and cannot be neglected in optical designs, and truncation, in general, increases the M^2 factor of the beam.

In Laser Modes, we will illustrate the higher-order eigensolutions to the propagation equation, and in The Propagation Constant, M^2 will be defined. The section Incorporating M^2 into the Propagation Equations defines how non-Gaussian beams propagate in free space and through optical systems.

LASER MODES

The fundamental TEM_{00} mode is only one of many transverse modes that satisfy the round-trip propagation criteria described in Gaussian Beam Propagation. Figure 2.13 shows examples of the primary lower-order Hermite-Gaussian (rectangular) solutions to the propagation equation.

Note that the subscripts n and m in the eigenmode TEM_{nm} are correlated to the number of nodes in the x and y directions. In each case, adjacent lobes of the mode are 180 degrees out of phase.

The propagation equation can also be written in cylindrical form in terms of radius (ρ) and angle (ϕ). The eigenmodes ($E_{\rho\phi}$) for this equation are a series of axially symmetric modes, which, for stable resonators, are closely approximated by Laguerre-Gaussian functions, denoted by $TEM_{\rho\phi}$. For the lowest-order mode, TEM_{00}, the Hermite-Gaussian and Laguerre-Gaussian functions are identical, but for higher-order modes, they differ significantly, as shown in figure 2.14.

The mode, TEM_{01}, also known as the "bagel" or "doughnut" mode, is considered to be a superposition of the Hermite-Gaussian TEM_{10} and TEM_{01} modes, locked in phase quadrature. In real-world lasers, the Hermite-Gaussian modes predominate since strain, slight misalignment, or contamination on the optics tends to drive the system toward rectangular coordinates. Nonetheless, the Laguerre-Gaussian TEM_{10} "target" or "bulls-eye" mode is clearly observed in well-aligned gas-ion and helium neon lasers with the appropriate limiting apertures.

THE PROPAGATION CONSTANT

The propagation of a pure Gaussian beam can be fully specified by either its beam waist diameter or its far-field divergence. In principle, full characterization of a beam can be made by simply measuring the waist diameter, $2w_0$, or by measuring the diameter, $2w(z)$, at a known and specified distance (z) from the beam waist, using the equations

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

and

$$R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

where λ is the wavelength of the laser radiation, and $w(z)$ and $R(z)$ are the beam radius and wavefront radius, respectively, at distance z from the beam waist. In practice, however, this approach is fraught with problems—it is extremely difficult, in many instances, to locate the beam waist; relying on a single-point measurement is inherently inaccurate; and, most important, pure Gaussian laser beams do not exist in the real world. The beam from a well-controlled helium neon laser comes very close, as does the beam from a few other gas lasers. However, for most lasers (even those

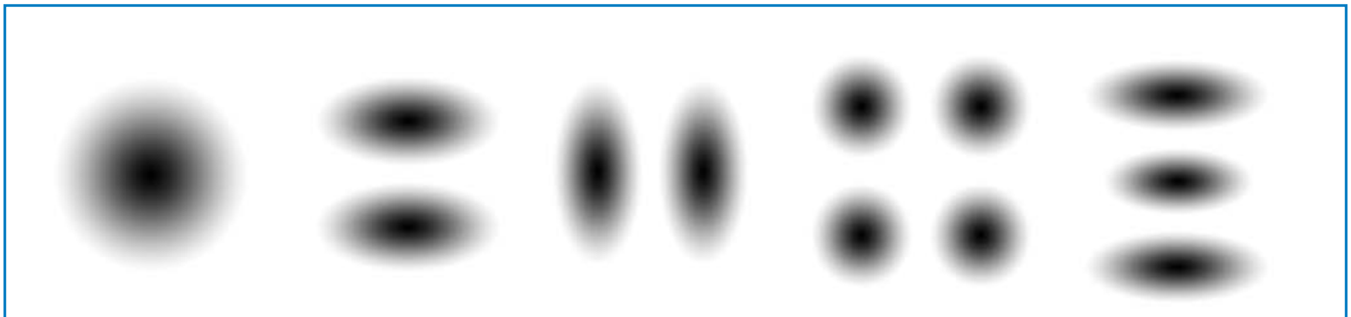

Figure 2.13 **Low-order Hermite-Gaussian resonator modes**

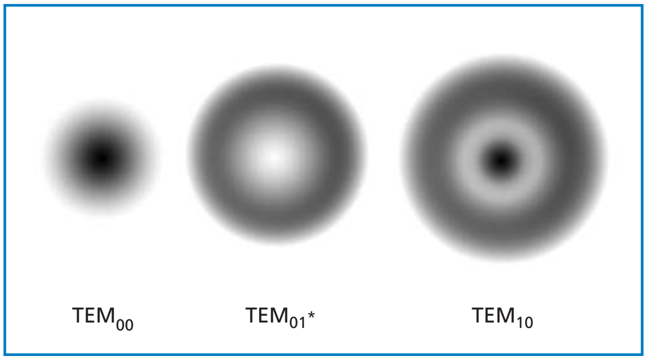

Figure 2.14 **Low-order axisymmetric resonator modes**

specifying a fundamental TEM_{00} mode), the output contains some component of higher-order modes that do not propagate according to the formula shown above. The problems are even worse for lasers operating in high-order modes.

The need for a figure of merit for laser beams that can be used to determine the propagation characteristics of the beam has long been recognized. Specifying the mode is inadequate because, for example, the output of a laser can contain up to 50 percent higher-order modes and still be considered TEM_{00}.

The concept of a dimensionless beam propagation parameter was developed in the early 1970s to meet this need, based on the fact that, for any given laser beam (even those not operating in the TEM_{00} mode) the product of the beam waist radius (w_0) and the far-field divergence (θ) are constant as the beam propagates through an optical system, and the ratio

$$M^2 = \frac{w_{0R}\,\theta_R}{w_0\,\theta} \qquad (2.25)$$

where w_{0R} and θ_R, the beam waist and far-field divergence of the real beam, respectively, is an accurate indication of the propagation characteristics of the beam. For a true Gaussian beam, $M^2 = 1$.

EMBEDDED GAUSSIAN

The concept of an "embedded Gaussian," shown in figure 2.15, is useful as a construct to assist with both theoretical modeling and laboratory measurements.

A mixed-mode beam that has a waist M (not M^2) times larger than the embedded Gaussian will propagate with a divergence M times greater than the embedded Gaussian. Consequently the beam diameter of the mixed-mode beam will always be M times the beam diameter of the embedded Gaussian, but it will have the same radius of curvature and the same Rayleigh range ($z = R$).

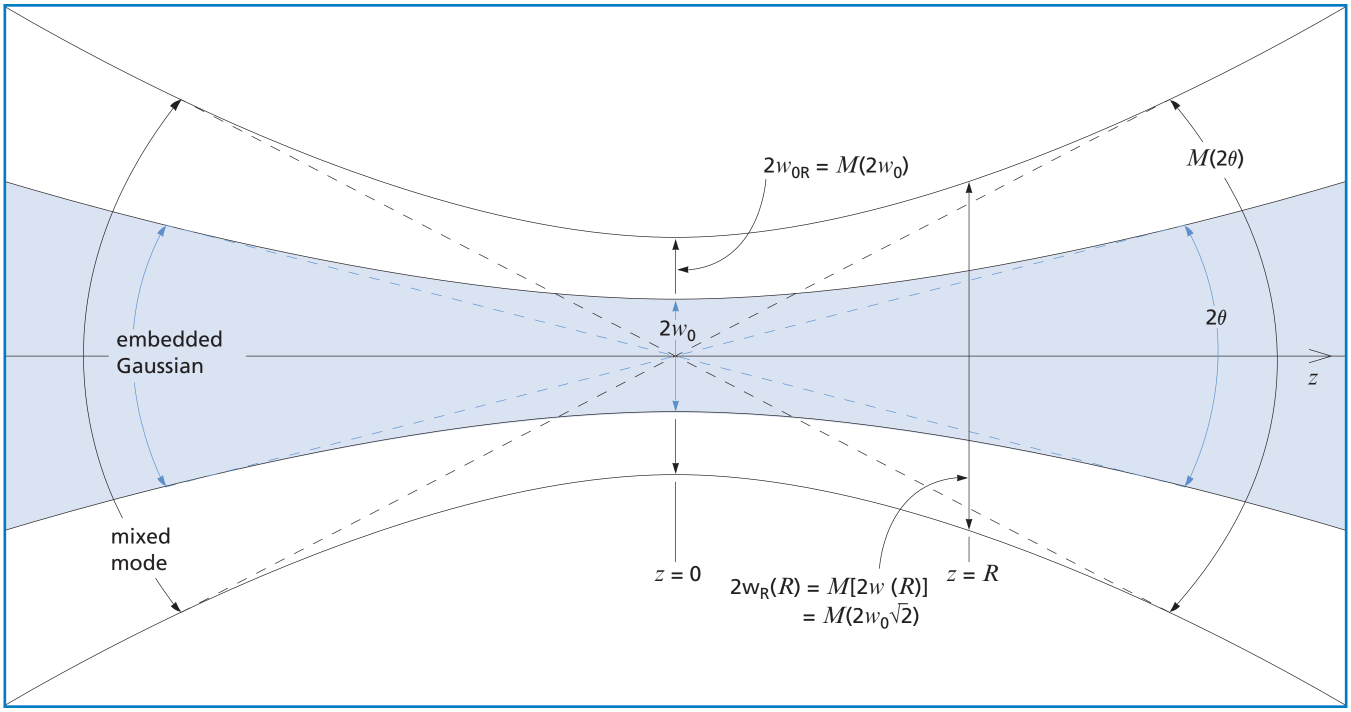

Figure 2.15 **The embedded Gaussian**

Incorporating M^2 Into the Propagation Equations

In the previous section we defined the propagation constant M^2

$$M^2 = \frac{w_{0R}\theta_R}{w_0\theta}$$

where w_{0R} and θ_R are the beam waist and far-field divergence of the real beam, respectively.

For a pure Gaussian beam, $M^2 = 1$, and the beam-waist beam-divergence product is given by

$$w_0\theta = \lambda/\pi$$

It follows then that for a real laser beam,

$$w_{0R}\theta_R = \frac{M^2\lambda}{\pi} > \frac{\lambda}{\pi} \tag{2.26}$$

The propagation equations for a real laser beam are now written as

$$w_R(z) = w_{0R}\left[1+\left(\frac{z\lambda M^2}{\pi w_{0R}^2}\right)^2\right]^{1/2} \tag{2.27}$$

and

$$R_R(z) = z\left[1+\left(\frac{\pi w_{0R}^2}{z\lambda M^2}\right)^2\right] \tag{2.28}$$

where $w_R(z)$ and $R_R(z)$ are the $1/e^2$ intensity radius of the beam and the beam wavefront radius at z, respectively.

The equation for w_0(optimum) now becomes

$$w_0(\text{optimum}) = \left(\frac{\lambda z M^2}{\pi}\right)^{1/2} \tag{2.29}$$

The definition for the Rayleigh range remains the same for a real laser beam and becomes

$$z_R = \frac{\pi w_{0R}^2}{M^2\lambda} \tag{2.30}$$

For $M^2 = 1$, these equations reduce to the Gaussian beam propagation equations .

$$R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

and

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

In a like manner, the lens equation can be modified to incorporate M^2. The standard equation becomes

$$\frac{1}{s+\left(z_R/M^2\right)^2/(s-f)} + \frac{1}{s''} = \frac{1}{f} \tag{2.31}$$

and the normalized equation transforms to

$$\frac{1}{(s/f)+\left(z_R/M^2 f\right)^2/(s/f-1)} + \frac{1}{(s''/f)} = 1. \tag{2.32}$$