Propagation Characteristics of Laser Beams

BEAM WAIST AND DIVERGENCE

Diffraction causes light waves to spread transversely as they propagate, and it is therefore impossible to have a perfectly collimated beam. The spreading of a laser beam is in accord with the predictions of diffraction theory. Under ordinary circumstances, the beam spreading can be so small it can go unnoticed. The following formulas accurately describe beam spreading, making it easy to see the capabilities and limitations of laser beams. The notation is consistent with much of the laser literature, particularly with Siegman's excellent *Lasers* (University Science Books).

Even if a Gaussian TEM_{00} laser-beam wavefront were made perfectly flat at some plane, with all rays there moving in precisely parallel directions, it would acquire curvature and begin spreading in accordance with

$$R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right] \quad (10.7)$$

and

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2} \quad (10.8)$$

where z is the distance propagated from the plane where the wavefront is flat, λ is the wavelength of light, w_0 is the radius of the $1/e^2$ irradiance contour at the plane where the wavefront is flat, $w(z)$ is the radius of the $1/e^2$ contour after the wave has propagated a distance z, and $R(z)$ is the wavefront radius of curvature after propagating a distance z. $R(z)$ is infinite at $z = 0$, passes through a minimum at some finite z, and rises again toward infinity as z is further increased, asymptotically approaching the value of z itself.

The plane $z = 0$ marks the location of a beam waist, or a place where the wavefront is flat, and w_0 is called the beam waist radius.

The irradiance distribution of the Gaussian TEM_{00} beam, namely,

$$I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2}, \quad (10.9)$$

where $w = w(z)$ and P is the total power in the beam, is the same at all cross sections of the beam. The invariance of the form of the distribution is a special consequence of the presumed Gaussian distribution at $z = 0$. Simultaneously, as $R(z)$ asymptotically approaches z for large z, $w(z)$ asymptotically approaches the value

$$w(z) = \frac{\lambda z}{\pi w_0} \quad (10.10)$$

where z is presumed to be much larger than $\pi w_0^2/\lambda$ so that the $1/e^2$ irradiance contours asymptotically approach a cone of angular radius

$$\theta = \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}. \quad (10.11)$$

This value is the far-field angular radius (half-angle divergence) of the Gaussian TEM_{00} beam. The vertex of the cone lies at the center of the waist (see figure 10.6).

It is important to note that, for a given value of λ, variations of beam diameter and divergence with distance z are functions of a single parameter, w_0, the beam waist radius.

NEAR-FIELD VS. FAR-FIELD DIVERGENCE

Unlike conventional light beams, Gaussian beams do not diverge linearly, as can be seen in figure 10.6. Near the laser, the divergence angle is extremely small; far from the laser, the divergence angle approaches the asymptotic limit described in equation 10.11 above. The Raleigh range (z_R), defined as the distance over which the beam radius spreads by a factor of $\sqrt{2}$, is given by

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (10.12)$$

The Raleigh range is the dividing line between near-field divergence and mid-range divergence. Far-field divergence (the number quoted in laser specifications) must be measured at a point $>z_R$ (usually $10z_R$ will suffice). This is a very important distinction because calculations for spot size and other parameters in an optical train will be inaccurate if near- or mid-field divergence values are used. For a tightly focused beam, the distance from the waist (the focal point) to the far field can be a few millimeters or less. For beams coming directly from the laser, the far-field distance can be measured in meters.

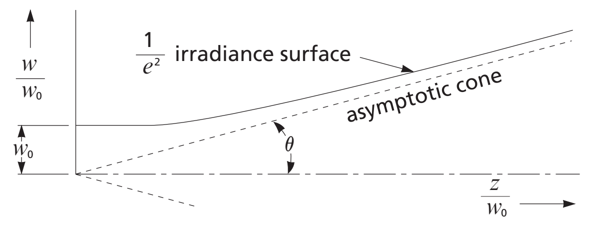


Figure 10.6 **Growth in beam diameter as a function of distance from the beam waist**

LOCATING THE BEAM WAIST

For a Gaussian laser beam, the location (and radius) of the beam waist is determined uniquely by the radius of curvature and optical spacing of the laser cavity mirrors because, at the reflecting surfaces of the cavity mirrors, the radius of curvature of the propagating beam is exactly the same as that of the mirrors. Consequently, for the flat/curved cavity shown in figure 10.7 (a), the beam waist is located at the surface of the flat mirror. For a symmetric cavity (b), the beam waist is halfway between the mirrors; for non-symmetric cavities (c and d), the beam waist is located by using the equation

$$z_1 = \frac{L(R_2 - L)}{R_1 + R_2 - 2L} \qquad (10.13)$$

and

$$z_1 + z_2 = L$$

where L is the effective mirror spacing, R_1 and R_2 are the radii of curvature of the cavity mirrors, and z_1 and z_2 are the distances from the beam waist of mirrors 1 and 2, respectively. (Note that distances are measured from the beam waist, and that, by convention, mirror curvatures that are concave when viewed from the waist are considered positive, while those that are convex are considered negative.)

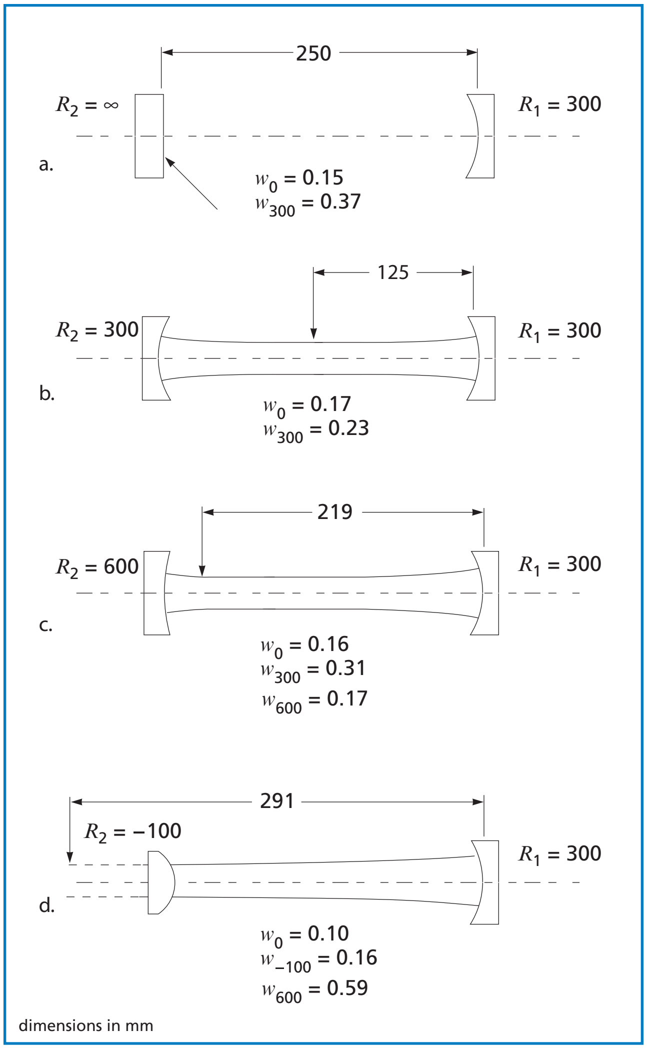


Figure 10.7 **Location of beam waist for common cavity geometries**

In any case but that of a flat output mirror, the beam waist is refracted as it passes through the mirror substrate. If the output coupler's second surface is flat, the effective waist of the refracted beam is moved toward the output coupler and is reduced in diameter. However, by applying a spherical correction to the second surface of the output coupler, the location of the beam waist can be moved to the output coupler itself, increasing the beam waist diameter and reducing far-field divergence. (See Calculating a Correcting Surface.)

It is useful, particularly when designing laser cavities, to understand the effect that mirror spacing has on the beam radius, both at the waist and at the curved mirror. Figure 10.8 plots equations 10.7 and 10.8 as a function of R/z (curved mirror radius divided by the mirror spacing). As the mirror spacing approaches the radius of curvature of the mirror ($R/z = 1$), the beam waist decreases dramatically, and the beam radius at the curved mirror becomes very large. On the other hand, as R/z becomes large, the beam radius at the waist and at the curved mirror are approximately the same.

CALCULATING A CORRECTING SURFACE

A laser beam is refracted as it passes through a curved output mirror. If the mirror has a flat second surface, the waist of the refracted beam moves closer to the mirror, and the divergence is increased. To counteract this, laser manufacturers often put a radius on the output coupler's second surface to collimate the beam by making a waist at the output coupler.

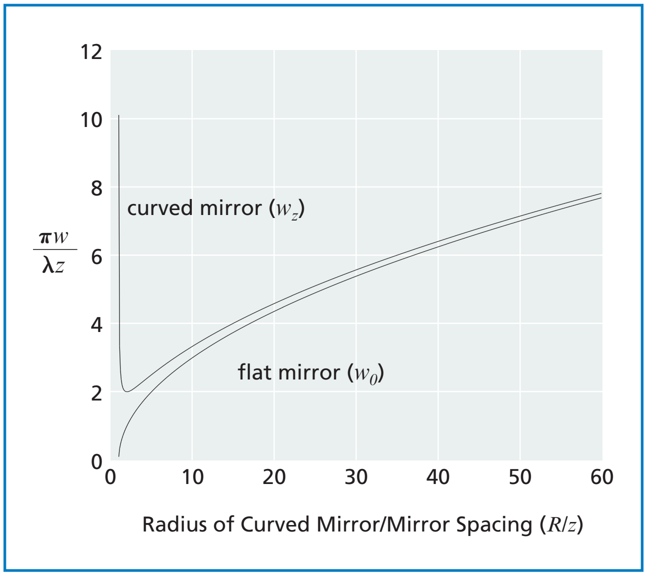


Figure 10.8 **Beam waist and output diameter as a function of mirror radius and separation**

This is illustrated by the case of a typical helium neon laser cavity consisting of a flat high reflector and an output mirror with a radius of curvature of 20 cm separated by 15 cm. If the laser is operating at 633 nm, the beam waist radius, beam radius at the output coupler, and beam half-angle divergence are

$$w_0 = 0.13 \text{ mm}, \; w_{200} = 0.26 \text{ mm}, \; \text{and } \theta = 1.5 \text{ mrad},$$

respectively; however, with a flat second surface, the divergence nearly doubles to 2.8 mrad. Geometrical optics would give the focal length of the lens formed by the correcting output coupler as 15 cm; a rigorous calculation using Gaussian beam optics shows it should be 15.1 cm. Using the lens-makers formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (10.14)$$

with the appropriate sign convention and assuming that $n = 1.5$, we get a convex correcting curvature of approximately 5.5 cm. At this point, the beam waist has been transferred to the output coupler, with a radius of 0.26 mm, and the far-field half-angle divergence is reduced to 0.76 mrad, a factor of nearly 4.

Correcting surfaces are used primarily on output couplers whose radius of curvature is a meter or less. For longer radius output couplers, the refraction effects are less dramatic, and a correcting second surface radius is unnecessary.

HIGHER ORDER GAUSSIAN LASER BEAMS

In the real world, the truly 100-percent, single transverse mode, Gaussian laser beam (also called a pure or fundamental mode beam) described by equations 10.7 and 10.8 is very hard to find. Low-power beams from helium neon lasers can be a close approximation, but the higher the power of the laser, and the more complex the excitation mechanism (e.g., transverse discharges, flash-lamp pumping), or the higher the order of the mode, the more the beam deviates from the ideal.

To address the issue of higher order Gaussian beams and mixed mode beams, a beam quality factor, M^2, has come into general use. A mixed mode is one where several modes are oscillating in the resonator at the same time. A common example is the mixture of the lowest order single transverse mode with the doughnut mode, before the intracavity mode limiting aperture is critically set to select just the fundamental mode. Because all beams have some wavefront defects, which implies they contain at least a small admixture of some higher order modes, a mixed mode beam is also called a "real" laser beam.

For a theoretical single transverse mode Gaussian beam, the value of the waist radius–divergence product is (from equation 10.11):

$$w_0\theta = \lambda / \pi. \qquad (10.15)$$

It is important to note that this product is an invariant for transmission of a beam through any normal, high-quality optical system (one that does not add aberrations to the beam wavefront). That is, if a lens focuses the single mode beam to a smaller waist radius, the convergence angle coming into the focus (and the divergence angle emerging from it) will be larger than that of the unfocused beam in the same ratio that the focal spot diameter is smaller: the product is invariant.

For a real laser beam, we have

$$W_0\Theta = M^2\lambda / \pi \qquad (10.16)$$

where W_0 and Θ are the $1/e^2$ intensity waist radius and the far-field half-divergence angle of the real laser beam, respectively. Here we have introduced the convention that upper case symbols are used for the mixed mode and lower case symbols for the fundamental mode beam coming from the same resonator. The mixed-mode beam radius W is M times larger than the fundamental mode radius at all propagation distances. Thus the waist radius is that much larger, contributing the first factor of M in equation 10.16. The second factor of M comes from the half-angle divergence, which is also M times larger. The waist radius–divergence half-angle product for the mixed mode beam also is an invariant, but is M^2 larger. The fundamental mode beam has the smallest divergence allowed by diffraction for a beam of that waist radius. The factor M^2 is called the "times-diffraction-limit" number or (inverse) beam quality; a diffraction-limited beam has an M^2 of unity.

For a typical helium neon laser operating in TEM_{00} mode, $M^2 < 1.05$. Ion lasers typically have an M^2 factor ranging from 1.1 to 1.7. For high-energy multimode lasers, the M^2 factor can be as high as 30 or 40. The M^2 factor describes the propagation characteristics (spreading rate) of the laser beam. It cannot be neglected in the design of an optical train to be used with the beam. Truncation (aperturing) by an optic, in general, increases the M^2 factor of the beam.

The propagation equations (analogous to equations 10.7 and 36.8) for the mixed-mode beam $W(z)$ and $R(z)$ are as follows:

$$W(z) = W_0\left[1 + \left(\frac{zM^2\lambda}{\pi W_0^2}\right)^2\right]^{1/2} = W_0\left[1 + \left(\frac{z}{Z_R}\right)^2\right] \qquad (10.17)$$

and

$$R(z) = z\left[1 + \left(\frac{\pi W_0^2}{zM^2\lambda}\right)^2\right] = z\left[1 + \left(\frac{Z_R}{z}\right)^2\right]. \qquad (10.18)$$

The Rayleigh range remains the same for a mixed mode laser beam:

$$Z_R = \frac{\pi W_0^2}{M^2\lambda} = \frac{\pi w_0^2}{\lambda} = z_R. \qquad (10.19)$$

Now consider the consequences in coupling a high M^2 beam into a fiber. Fiber coupling is a task controlled by the product of the focal diameter ($2W_f$) and the focal convergence angle (θ_f). In the tight focusing limit, the focal diameter is proportional to the focal length f of the lens, and is inversely proportional to the diameter of the beam at the lens (i.e., $2W_f \propto f/D_{lens}$).

The lens-to-focus distance is f, and, since $f \times \theta_f$ is the beam diameter at distance f in the far field of the focus, $D_{lens} \propto f\theta_f$. Combining these proportionalities yields

$$W_f\theta_f = \text{constant}$$

for the fiber-coupling problem as stated above. The diameter-divergence product for the mixed-mode beam is M^2 larger than the fundamental mode beam in accordance with equations 10.15 and 10.16.

There is a threefold penalty associated with coupling a beam with a high M^2 into a fiber: 1) the focal length of the focusing lens must be a factor of $1/M^2$ shorter than that used with a fundamental-mode beam to obtain the same focal diameter at the fiber; 2) the numerical aperture (NA) of the focused beam will be higher than that of the fundamental beam (again by a factor of $1/M^2$) and may exceed the NA of the fiber; and 3) the depth of focus will be smaller by $1/M^2$ requiring a higher degree of precision and stability in the optical alignment.

APPLICATION NOTE

Stable vs Unstable Resonator Cavities

A stable resonator cavity is defined as one that self-focuses energy within the cavity back upon itself to create the typical Gaussian modes found in most traditional lasers. The criterion for a stable cavity is that

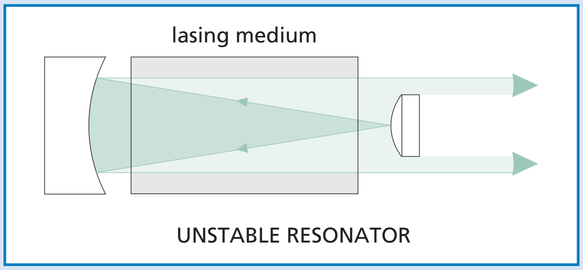
$$0 \le g_1 g_2 \le 1$$

where

$$g_1 = 1 - \frac{L}{R_1} \text{ and } g_2 = 1 - \frac{L}{R_2}$$

where R_1 and R_2 are the radii of the cavity mirrors and L is the mirror separation.

The mode volumes of stable resonator cavities are relatively small. This is fine when the excitation regions of a laser are also relatively small, as is the case with a HeNe or DPSS laser. However, for large-format high-energy industrial lasers, particularly those with high single-pass gain, stable resonators can limit the output. In these cases, unstable resonators, like the one shown in the illustration below, can generate higher output with better mode quality. In this case, the output coupling is determined by the ratio of the diameters of the output and high-reflecting mirrors, not the coating reflectivity. In the near field, the output looks like a doughnut, because the center of the beam is occluded by the output mirror. At a focus, however, the beam has most of the propagation characteristics of a fundamental-mode stable laser.



Unstable Resonator Design