Polarization

Polarization States

Four numbers are required to describe a single plane wave Fourier component traveling in the $+z$ direction. These can be thought of as the amplitude and phase shift of the field along two orthogonal directions.

1. CARTESIAN REPRESENTATION

In Cartesian coordinates, the propagation equation for an electric field is given by the formula

$$E(x,y,z) = (xE_x e^{i\phi_x} + yE_y e^{i\phi_y})e^{i(kz-\omega t)} \qquad (1.75)$$

where E_x, E_y, ϕ_x, and ϕ_y are real numbers defining the magnitude and the phase of the field components in the orthogonal unit vectors x and y. If the origin of time is irrelevant, only the relative phase shift

$$\phi = \phi_x - \phi_y \qquad (1.76)$$

need be specified.

2. CIRCULAR REPRESENTATION

In the circular representation, we resolve the field into circularly polarized components. The basic states are represented by the complex unit vectors

$$e_+ = \left(1/\sqrt{2}\right)\left(x+iy\right) \text{ and} \qquad (1.77)$$

$$e_- = \left(1/\sqrt{2}\right)\left(x-iy\right) \qquad (1.78)$$

where e_+ is the unit vector for left circularly polarized light; for positive helicity light; for light that rotates counterclockwise in a fixed plane as viewed facing into the light wave; and for light whose electric field rotation obeys the right hand rule with thumb pointing in the direction of propagation.

APPLICATION NOTE

Polarization Convention

Historically, the orientation of a polarized electromagnetic wave has been defined in the optical regime by the orientation of the electric vector. This is the convention used by CVI Melles Griot.

e_- is the unit vector for right circularly polarized light; for negative helicity light; for light that rotates clockwise in a fixed plane as viewed facing into the light wave; and for light whose electric field rotation disobeys the right hand rule with thumb pointing in the direction of propagation.

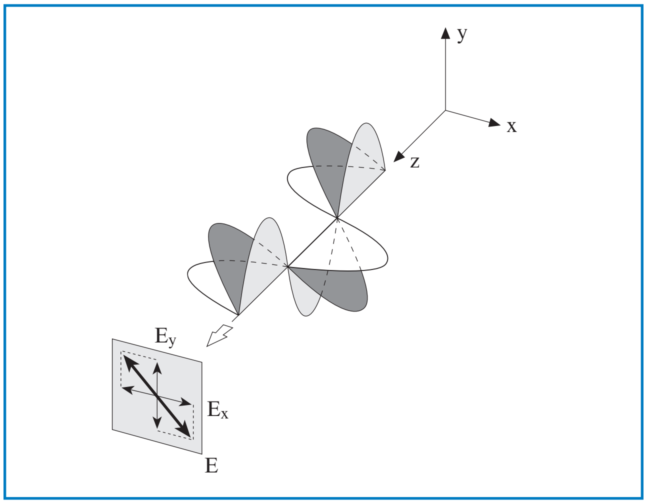

Figure 1.47 **Linearly polarized light. E_x and E_y are in phase**

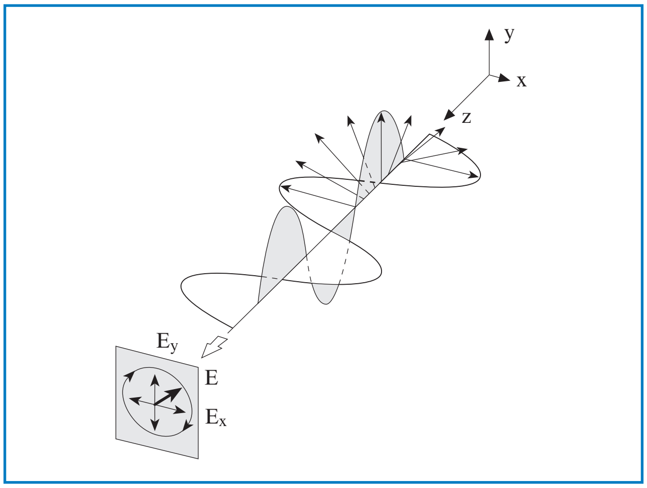

Figure 1.48 **Circularly polarized light. E_x and E_y are out of phase by angular frequency ω**

As in the case of the Cartesian representation, we write

$$E=(e_{+}E_{+}e^{i\phi+} + e_{-}E_{-}e^{i\phi-})\, e^{i(kz-\omega t)} \qquad (1.79)$$

where E_{+}, E_{-}, ϕ_{+}, and ϕ_{-} are four real numbers describing the magnitudes and phases of the field components of the left and right circularly polarized components. Note that

$$E_{+}=e_{-}\bullet E \qquad (1.80)$$

$$E_{-}=e_{+}\bullet E \qquad (1.81)$$

3. ELLIPTICAL REPRESENTATION

An arbitrary polarization state is generally elliptically polarized. This means that the tip of the electric field vector will describe an ellipse, rotating once per optical cycle.

Let a be the semimajor and b be the semiminor axis of the polarization ellipse. Let ψ be the angle that the semimajor axis makes with the x axis.

Let ξ and η be the axes of a right-handed coordinate system rotated by an angle $+\psi$ with respect to the x axis and aligned with the polarization ellipse as shown in Figure 1.49.

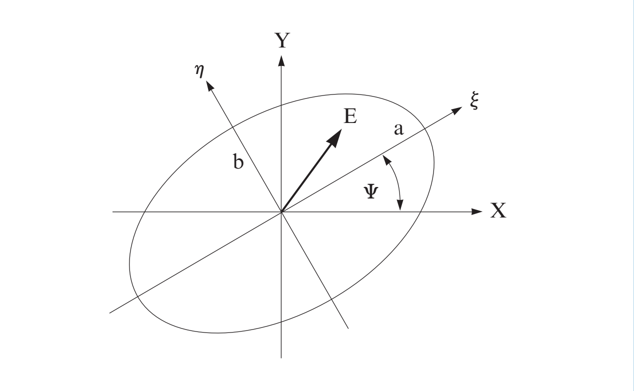

Figure 1.49 **The polarization ellipse**

The elliptical representation is:

$$E=\left(a\hat{\xi}+b\hat{\eta}\right)e^{i\delta_0}e^{i(kz-\omega t)} \qquad (1.82)$$

Note that the phase shift δ_0 above is required to adjust the time origin, and the parameter ψ is implicit in the rotation of the ξ and η axes with respect to the x and y axes.

CONVERSIONS BETWEEN REPRESENTATIONS

For brevity, we will provide only the Cartesian to circular and Cartesian to elliptical transformations. The inverse transformations are straightforward. We define the following quantities:

$$g_1=E_x\cos\phi_x-E_y\sin\phi_y \qquad (1.83)$$

$$g_2=E_x\sin\phi_x+E_y\cos\phi_y \qquad (1.84)$$

$$g_3=E_x\cos\phi_x+E_y\sin\phi_y \qquad (1.85)$$

$$g_4=E_x\sin\phi_x-E_y\cos\phi_y \qquad (1.86)$$

$$u=\left(g_1^2+g_2^2\right)^{1/2} \qquad (1.87)$$

$$v=\left(g_3^2+g_4^2\right)^{1/2} \qquad (1.88)$$

$$\phi_{12}=\text{atan}\,(g_1,g_2) \qquad (1.89)$$

$$\phi_{34}=\text{atan}\,(g_3,g_4) \qquad (1.90)$$

In the above, atan(x,y) is the four quadrant arc tangent function. This means that atan$(x,y)=$ atan(y/x) with the provision that the quadrant of the angle returned by the function is controlled by the signs of both x and y, not just the sign of their quotient; for example, if $g_2=g_1=-1$, then ϕ_{12} above is $5\pi/4$ or $-3\pi/4$, not $\pi/4$.

A. CARTESIAN TO CIRCULAR TRANSFORMATION

$$E_{+}=v/\sqrt{2} \qquad (1.91)$$

$$E_{-}=u/\sqrt{2} \qquad (1.92)$$

$$\phi_{+}=\phi_{34} \qquad (1.93)$$

$$\phi_{-}=\phi_{12} \qquad (1.94)$$

B. CARTESIAN TO ELLIPTICAL TRANSFORMATION

$$a=(v+u)/2 \qquad (1.95)$$

$$b=(v-u)/2 \qquad (1.96)$$

$$\psi=(\phi_{12}-\phi_{34})/2 \qquad (1.97)$$

$$\delta_0=(\phi_{12}+\phi_{34})/2 \qquad (1.98)$$

LINEAR POLARIZERS

A linear polarizer is a device that creates a linear polarization state from an arbitrary input. It does this by removing the component orthogonal to the selected state. Unlike plastic sheet polarizers which absorb the rejected beam (which turns into heat), cube polarizers and thin-film plate polarizers reflect the rejected beam, creating two usable beams. Still others may refract the two polarized beams at different angles, thereby separating them. Examples are Wollaston and Rochon prism polarizers.

Suppose the pass direction of the polarizer is determined by unit vector p. Then the transmitted field E_2, in terms of the incident field E_1, is given by

$$E_2 = p(p \bullet E_1) \quad (1.99)$$

where the phase shift of the transmitted field has been ignored.

A real polarizer has a pass transmission, $T_{||}$, less than 1. The transmission of the rejected beam, $T_{\perp}$, may not be 0. If r is a unit vector along the rejected direction, then

$$E_2 = (T_{||})^{1/2} p(p \bullet E_1)e^{i\phi_{||}} + (T_{\perp})^{1/2} r(r \bullet E_1)e^{i\phi_{\perp}} \quad (1.100)$$

In the above, the phase shifts along the two directions must be retained. Similar expressions could be arrived at for the rejected beam. If θ is the angle between the field E_1 and the polarizer pass direction p, the above equation predicts that

$$T = T_{||}\cos^2\theta + T_{\perp}\sin^2\theta \quad (1.101)$$

The above equation shows that, when the polarizer is aligned so that $\theta = 0$, $T = T_{||}$. When it is "crossed", $\theta = \pi/2$, and $T = T_{\perp}$. The extinction ratio is $\epsilon = T_{||} / T_{\perp}$. A polarizer with perfect extinction has $T_{\perp} = 0$, and thus $T = T_{||}\cos^2\theta$ is a familiar result. Because $\cos^2\theta$ has a broad maximum as a function of orientation angle, setting a polarizer at a maximum of transmission is generally not very accurate. One has to either map the $\cos^2\theta$ with sufficient accuracy to find the $\theta = 0$ point, or do a null measurement at $\theta = \pm\pi/2$.

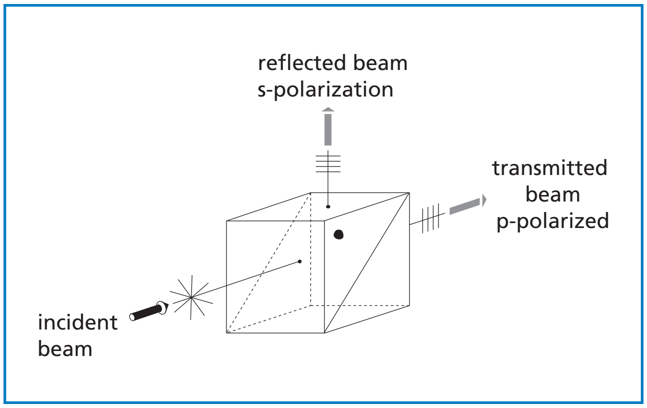

Figure 1.50 **At CVI Melles Griot, the DOT marks preferred input face. This is the tested direction for transmitted wavefront. Damage threshold is also higher for this orientation as well.**

Polarization Definitions

BIREFRINGENCE

A birefringent crystal, such as calcite, will divide an entering beam of monochromatic light into two beams having opposite polarization. The beams usually propagate in different directions and will have different speeds. There will be only one or two optical axis directions within the crystal in which the beam will remain collinear and continue at the same speed, depending on whether the birefringent crystal is uniaxial or biaxial.

If the crystal is a plane-parallel plate, and the optical axis directions are not collinear with the beam, radiation will emerge as two separate, orthogonally polarized beams (see Figure 1.51). The beam will be unpolarized where the beams overlap upon emergence. The two new beams within the material are distinguished from each other by more than just polarization and velocity. The rays are referred to as extraordinary (E) and ordinary (O). These rays need not be confined to the plane of incidence. Furthermore, the velocity of these rays changes with direction. Thus, the index of refraction for extraordinary rays is also a continuous function of direction. The index of refraction for the ordinary ray is constant and is independent of direction.

The two indexes of refraction are equal only in the direction of an optical axis within the crystal. The dispersion curve for ordinary rays is a single, unique curve when the index of refraction is plotted against wavelength. The dispersion curve for the extraordinary ray is a family of curves with different curves for different directions. Unless it is in a particular polarization state, or the crystalline surface is perpendicular to an optical axis, a ray normally incident on a birefringent surface will be divided in two at the boundary. The extraordinary ray will be deviated; the ordinary ray will not. The ordinary ray index n, and the most extreme (whether greater or smaller) extraordinary ray index n_e, are together known as the principal indices of refraction of the material.

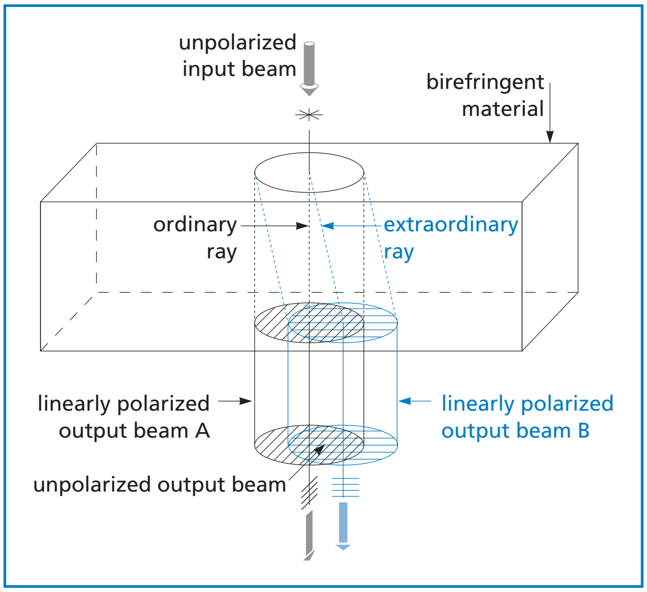

Figure 1.51 **Double refraction in a birefringent crystal**

If a beam of linearly polarized monochromatic light enters a birefringent crystal along a direction not parallel to the optical axis of the crystal, the beam will be divided into two separate beams. Each will be polarized at right angles to the other and will travel in different directions. The original beam energy, which will be divided between the new beams, depends on the original orientation of the vector to the crystal.

The energy ratio between the two orthogonally polarized beams can be any value. It is also possible that all energy will go into one of the new beams. If the crystal is cut as a plane-parallel plate, these beams will recombine upon emergence to form an elliptically polarized beam.

The difference between the ordinary and extraordinary ray may be used to create birefringent crystal polarization devices. In some cases, the difference in refractive index is used primarily to separate rays and eliminate one of the polarization planes, for example, in Glan-type polarizers. In other cases, such as Wollaston and Thompson beamsplitting prisms, changes in propagation direction are optimized to separate an incoming beam into two orthogonally polarized beams.

DICHROISM

Dichroism is selective absorption of one polarization plane over the other during transmission through a material. Sheet-type polarizers are manufactured with organic materials imbedded into a plastic sheet. The sheet is stretched, aligning molecules and causing them to be birefringent, and then dyed. The dye molecules selectively attach themselves to aligned polymer molecules, so that absorption is high in one plane and weak in the other. The transmitted beam is linearly polarized. Polarizers made of such material are very useful for low-power and visual applications. The usable field of view is large (up to grazing incidence), and diameters in excess of 100 mm are available.

POLARIZATION BY REFLECTION

When a beam of ordinary light is incident at the polarizing angle on a transmissive dielectric such as glass, the emerging refracted ray is partially linearly polarized. For a single surface (with n=1.50) at Brewster's angle, 100 percent of the light whose electric vector oscillates parallel to the plane of incidence is transmitted. Only 85 percent of the perpendicular light is transmitted (the other 15 percent is reflected). The degree of polarization from a single-surface reflection is small.

If a number of plates are stacked parallel and oriented at the polarizing angle, some vibrations perpendicular to the plane of incidence will be reflected at each surface, and all those parallel to it will be refracted. By making the number of plates within the stack large (more than 25), high degrees of linear polarization may be achieved. This polarization method is utilized in CVI Melles Griot polarizing beamsplitter cubes which are coated with many layers of quarter-wave dielectric thin films on the interior prism angle. This beamsplitter separates an incident laser beam into two perpendicular and orthogonally polarized beams.

THIN METAL FILM POLARIZERS

Optical radiation incident on small, elongated metal particles will be preferentially absorbed when the polarization vector is aligned with the long axis of the particle. CVI Melles Griot infrared polarizers utilize this effect to make polarizers for the near-infrared. These polarizers are considerably more effective than dichroic polarizers.

Polarizing thin films are formed by using the patented Slocum process to deposit multiple layers of microscopic silver prolate spheroids onto a polished glass substrate. The exact dimensions of these spheroids determine the optical properties of the film. Peak absorption can be selected for any wavelength from 400 to 3000 nm by controlling the deposition process. Contrast ratios up to 10,000:1 can be achieved with this method. Other CVI Melles Griot high-contrast polarizers exhibit contrasts as high as 100,000:1.

CALCITE

Calcite, a rhombohedral crystalline form of calcium carbonate, is found in various forms such as limestone and marble. Since calcite is a naturally occurring material, imperfections are not unusual. The highest quality materials, those that exhibit no optical defects, are difficult to find and are more expensive than those with some defects. Applications for calcite components typically fall into laser applications or optical research. CVI Melles Griot offers calcite components in two quality grades to meet those various needs.

There are three main areas of importance in defining calcite quality.

Spectral Properties

Trace amounts of chemical impurities, as well as lattice defects, can cause calcite to be colored, which changes absorption. For visible light applications, it is essential to use colorless calcite. For near-infrared applications, material with a trace of yellow is acceptable. This yellow coloration results in a 15-percent to 20-percent decrease in transmission below 420 nm.

Wavefront Distortion (Striae)

Striae, or streaked fluctuations in the refractive index of calcite, are caused by dislocations in the crystal lattice. They can cause distortion of a light wavefront passing through the crystal. This is particularly troublesome for interferometric applications.

Scatter

Small inclusions within the calcite crystal account for the main source of scatter. They may appear as small cracks or bubbles. In general, scatter presents a significant problem only when the polarizer is being used with a laser. The amount of scatter centers that can be tolerated is partially determined by beam size and power.

CVI MELLES GRIOT CALCITE GRADES

CVI Melles Griot has selected the most applicable calcite qualities, grouped into two grades:

Laser Grade

Calcite with a wavefront deformation of $\lambda/4$ at 633 nm or better due to striae only.

Optical Grade

Calcite with a wavefront deformation of 1λ to $\lambda/4$ at 633 nm due to striae only.