# Modulation Transfer Function

The modulation transfer function (MTF), a quantitative measure of image quality, is far superior to any classic resolution criteria. MTF describes the ability of a lens or system to transfer object contrast to the image. MTF plots can be associated with the subsystems that make up a complete electro-optical or photographic system. MTF data can be used to determine the feasibility of overall system expectations.

Bar-chart resolution testing of lens systems is deceptive because almost 20 percent of the energy arriving at a lens system from a bar chart is modulated at the third harmonic and higher frequencies. Consider instead a sine-wave chart in the form of a positive transparency in which transmittance varies in one dimension. Assume that the transparency is viewed against a uniformly illuminated background. The maximum and minimum transmittances are $T_{max}$ and $T_{min}$, respectively. A lens system under test forms a real image of the sine-wave chart, and the spatial frequency ($u$) of the image is measured in cycles per millimeter. Corresponding to the transmittances $T_{max}$ and $T_{min}$ are the image irradiances $I_{max}$ and $I_{min}$. By analogy with Michelson's definition of visibility of interference fringes, the contrast or modulation of the chart and image are defined, respectively, as

$$M_c = \frac{T_{max} - T_{min}}{T_{max} + T_{min}} \quad (3.1)$$

and

$$M_i = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (3.2)$$

where $M_c$ is the modulation of the chart and $M_i$ is the modulation of the image.

The modulation transfer function (MTF) of the optical system at spatial frequency $u$ is then defined to be

$$\mathrm{MTF} = \mathrm{MTF}(u) = M_i / M_c. \quad (3.3)$$

The graph of MTF versus $u$ is a modulation transfer function curve and is defined only for lenses or systems with positive focal length that form real images.

It is often convenient to plot the magnitude of MTF($u$) versus $u$. Changes in MTF curves are easily seen by graphical comparison. For example, for lenses, the MTF curves change with field angle positions and conjugate ratios. In a system with astigmatism or coma, different MTF curves are obtained that correspond to various azimuths in the image plane through a single image point. For cylindrical lenses, only one azimuth is meaningful. MTF curves can be either polychromatic or monochromatic. Polychromatic curves show the effect of any chromatic aberration that may be present. For a well-corrected achromatic system, polychromatic MTF can be computed by weighted averaging of monochromatic MTFs at a single image surface. MTF can also be measured by a variety of commercially available instruments. Most instruments measure polychromatic MTF directly.

## PERFECT CIRCULAR LENS

The monochromatic, diffraction-limited MTF (or MDMTF) of a circular aperture (perfect aberration-free spherical lens) at an arbitrary conjugate ratio is given by the formula

$$\mathrm{MDMTF}(x) = \frac{2}{\pi}\left[\arccos(x) - x\sqrt{1-x^2}\right] \quad (3.4)$$

where the arc cosine function is in radians and $x$ is the normalized spatial frequency defined by

$$x = \frac{u}{u_{ic}} \quad (3.5)$$

where $u$ is the absolute spatial frequency and $u_{ic}$ is the incoherent diffraction cutoff spatial frequency. There are several formulas for $u_{ic}$ including

$$u_{ic} = \frac{1.22}{r_d} = \frac{n''D\sqrt{1-\left(\frac{1.22\lambda}{n''D}\right)^2}}{\lambda s''} = \frac{2n''\sin(u'')\sqrt{1-\left(\frac{1.22\lambda}{n''D}\right)^2}}{\lambda} = \frac{2n''\sin(u'')}{\lambda} = \frac{n''D}{\lambda s''} \quad (3.6)$$

where $r_d$ is the linear spot radius in the case of pure diffraction (Airy disc radius), $D$ is the diameter of the lens clear aperture (or of a stop in near-contact), $\lambda$ is the wavelength, $s$ is the secondary conjugate distance, $u''$ is the largest angle between any ray and the optical axis at the secondary conjugate point, the product $n''$ sin($u''$) is by definition the image space numerical aperture, and $n''$ is the image space refractive index. It is essential that $D$, $\lambda$, and $s''$ have consistent units (usually millimeters, in which case $u$ and $u_{ic}$ will be in cycles per millimeter). The relationship

$$\sin(u'') = \frac{D}{2s''} \quad (3.7)$$

implies that the secondary principal surface is a sphere centered upon the secondary conjugate point. This means that the lens is completely free of spherical aberration and coma, and, in the special case of infinite conjugate ratio ($s'' = f''$),

$$u_{ic} = n''\frac{D}{\lambda f}. \quad (3.8)$$

## PERFECT RECTANGULAR LENS

The MDMTF of a rectangular aperture (perfect aberration-free cylindrical lens) at arbitrary conjugate ratio is given by the formula

$$\mathrm{MDMTF}(x) = (1 - x) \tag{3.9}$$

where $x$ is again the normalized spatial frequency $u/u_{ic}$, where, in the present cylindrical case,

$$u_{ic} = \frac{1}{r_d} \tag{3.10}$$

and $r_d$ is one-half the full width of the central stripe of the diffraction pattern measured from first maximum to first minimum. This formula differs by a factor of 1.22 from the corresponding formula in the circular aperture case. The following applies to both circular and rectangular apertures:

$$u_{ic} = \frac{2n'' \sin(u'')}{\lambda} \tag{3.11}$$

The remaining three expressions for $u_{ic}$ in the circular aperture case can be applied to the present rectangular aperture case provided that two substitutions are made. Everywhere the constant 1.22 formerly appeared, it must be replaced by 1.00. Also, the aperture diameter $D$ must now be replaced by the aperture width $w$. The relationship $\sin(u'') = w/2s''$ means that the secondary principal surface is a circular cylinder centered upon the secondary conjugate line. In the special case of infinite conjugate ratio, the incoherent cutoff frequency for cylindrical lenses is

$$u_{ic} = n'' \frac{w}{\lambda f} \tag{3.12}$$

## IDEAL PERFORMANCE AND REAL LENSES

In an ideal lens, the $x$-intercept and the MDMTF-intercept are at unity (1.0). MDMTF($x$) for the rectangular case is a straight line between these intercepts. For the circular case, MDMTF($x$) is a curve that dips slightly below the straight line. These curves are shown in figure 3.2. Maximum contrast (unity) is apparent when spatial frequencies are low (i.e., for large features). Poor contrast is apparent when spatial frequencies are high (i.e., small features). All examples are limited at high frequencies by diffraction effects. A normalized spatial frequency of unity corresponds to the diffraction limit.

All real cylindrical, monochromatic MTF curves fall on or below the straight MDMTF($x$) line. Similarly, all real spherical and monochromatic MTF curves fall on or below the circular MDMTF($x$) curve. Thus the two ideal MDMTF($x$) curves represent the perfect (ideal) optical performance. Optical element or system quality is measured by how closely the real MTF curve approaches the corresponding ideal MDMTF($x$) curve (see figure 3.3).

MTF is an extremely sensitive measure of image degradation. To illustrate this, consider a lens having a quarter wavelength of spherical aberration. This aberration, barely discernible by eye, would reduce the MTF by as much as 0.2 at the midpoint of the spatial frequency range.

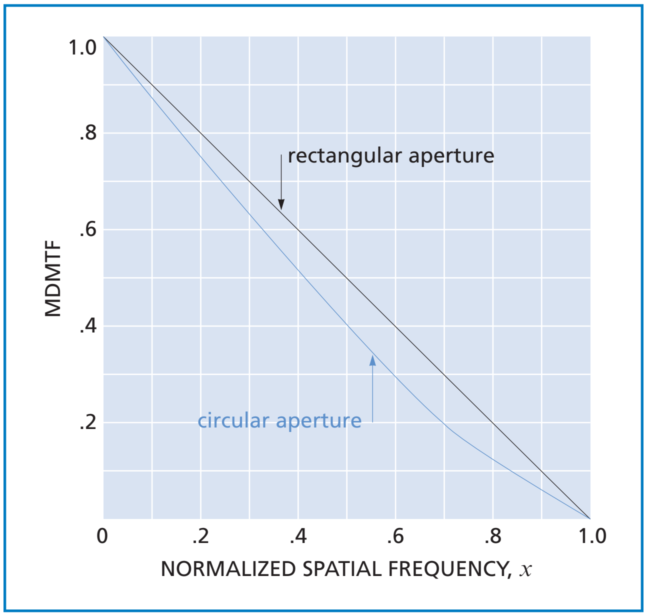


Figure 3.2 **MDMTF as a function of normalized spatial frequency**

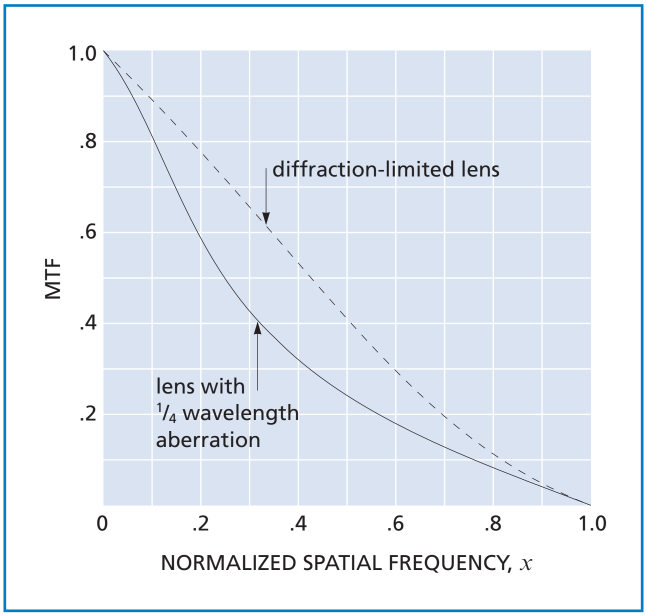


Figure 3.3 **MTF as a function of normalized spatial frequency**