Diffraction Effects

In all light beams, some energy is spread outside the region predicted by geometric propagation. This effect, known as diffraction, is a fundamental and inescapable physical phenomenon. Diffraction can be understood by considering the wave nature of light. Huygens' principle (figure 1.26) states that each point on a propagating wavefront is an emitter of secondary wavelets. The propagating wave is then the envelope of these expanding wavelets. Interference between the secondary wavelets gives rise to a fringe pattern that rapidly decreases in intensity with increasing angle from the initial direction of propagation. Huygens' principle nicely describes diffraction, but rigorous explanation demands a detailed study of wave theory.

Diffraction effects are traditionally classified into either Fresnel or Fraunhofer types. Fresnel diffraction is primarily concerned with what happens to light in the immediate neighborhood of a diffracting object or aperture. It is thus only of concern when the illumination source is close to this aperture or object, known as the near field. Consequently, Fresnel diffraction is rarely important in most classical optical setups, but it becomes very important in such applications as digital optics, fiber optics, and near-field microscopy.

Fraunhofer diffraction, however, is often important even in simple optical systems. This is the light-spreading effect of an aperture when the aperture (or object) is illuminated with an infinite source (plane-wave illumination) and the light is sensed at an infinite distance (far-field) from this aperture.

From these overly simple definitions, one might assume that Fraunhofer diffraction is important only in optical systems with infinite conjugate, whereas Fresnel diffraction equations should be considered at finite conjugate ratios. Not so. A lens or lens system of finite positive focal length with plane-wave input maps the far-field diffraction pattern of its aperture onto the focal plane; therefore, it is Fraunhofer diffraction that determines the limiting performance of optical systems. More generally, at any conjugate ratio, far-field angles are transformed into spatial displacements in the image plane.

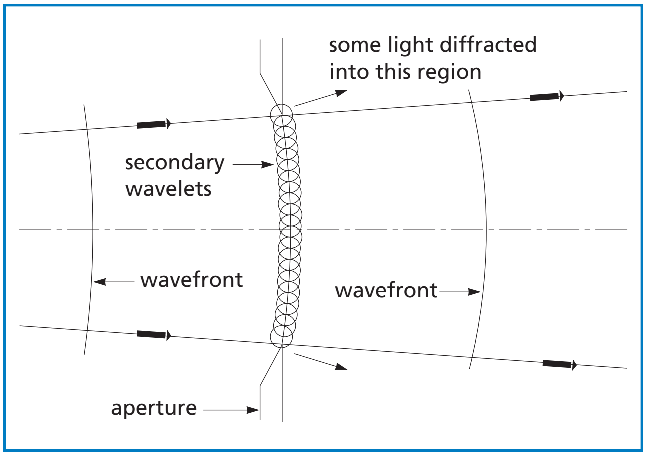

Figure 1.26 **Huygens' principle**

CIRCULAR APERTURE

Fraunhofer diffraction at a circular aperture dictates the fundamental limits of performance for circular lenses. It is important to remember that the spot size, caused by diffraction, of a circular lens is

$$d = 2.44\lambda(f/\#) \qquad (1.25)$$

where d is the diameter of the focused spot produced from plane-wave illumination and λ is the wavelength of light being focused. Notice that it is the f-number of the lens, not its absolute diameter, that determines this limiting spot size.

The diffraction pattern resulting from a uniformly illuminated circular aperture actually consists of a central bright region, known as the Airy disc (see figure 1.27), which is surrounded by a number of much fainter rings. Each ring is separated by a circle of zero intensity. The irradiance distribution in this pattern can be described by

$$I_x = I_0\left[\frac{2J_1(x)}{x}\right]^2 \qquad (1.26)$$

where

I_0 = peak irradiance in the image

$$J_1(x) = \text{Bessel function of the first kind of order unity}$$

$$= x\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n-2}}{(n-1)!n!2^{2n-1}}$$

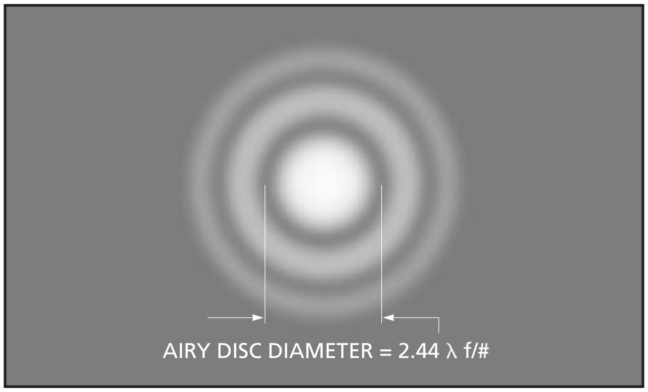

Figure 1.27 **Center of a typical diffraction pattern for a circular aperture**

and

$$x = \frac{\pi D}{\lambda}\sin\theta$$

where

λ = wavelength

D = aperture diameter

θ = angular radius from the pattern maximum.

This useful formula shows the far-field irradiance distribution from a uniformly illuminated circular aperture of diameter D.

SLIT APERTURE

A slit aperture, which is mathematically simpler, is useful in relation to cylindrical optical elements. The irradiance distribution in the diffraction pattern of a uniformly illuminated slit aperture is described by

$$I_x = I_0\left[\frac{\sin x}{x}\right]^2 \quad (1.27)$$

where I_0 = peak irradiance in image

$$x = \frac{\pi w \sin\theta}{\lambda}$$

where λ = wavelength

w = slit width

θ = angular deviation from pattern maximum.

APPLICATION NOTE

Rayleigh Criterion

In imaging applications, spatial resolution is ultimately limited by diffraction. Calculating the maximum possible spatial resolution of an optical system requires an arbitrary definition of what is meant by resolving two features. In the Rayleigh criterion, it is assumed that two separate point sources can be resolved when the center of the Airy disc from one overlaps the first dark ring in the diffraction pattern of the second. In this case, the smallest resolvable distance, d, is

$$d = \frac{0.61\lambda}{\text{NA}} = 1.22\lambda(\text{f/\#})$$

ENERGY DISTRIBUTION TABLE

The accompanying table shows the major features of pure (unaberrated) Fraunhofer diffraction patterns of circular and slit apertures. The table shows the position, relative intensity, and percentage of total pattern energy corresponding to each ring or band. It is especially convenient to characterize positions in either pattern with the same variable x. This variable is related to field angle in the circular aperture case by

$$\sin\theta = \frac{\lambda x}{\pi D} \quad (1.28)$$

where D is the aperture diameter. For a slit aperture, this relationship is given by

$$\sin\theta = \frac{\lambda x}{\pi w} \quad (1.29)$$

where w is the slit width, π has its usual meaning, and D, w, and λ are all in the same units (preferably millimeters). Linear instead of angular field positions are simply found from $r = s''\tan\theta$ where s'' is the secondary conjugate distance. This last result is often seen in a different form, namely the diffraction-limited spot-size equation, which, for a circular lens is

$$d = 2.44\,\lambda(\text{f/\#}) \quad \text{(see eq. 1.25)}$$

This value represents the smallest spot size that can be achieved by an optical system with a circular aperture of a given f-number, and it is the diameter of the first dark ring, where the intensity has dropped to zero.

The graph in figure 1.28 shows the form of both circular and slit aperture diffraction patterns when plotted on the same normalized scale. Aperture diameter is equal to slit width so that patterns between x-values and angular deviations in the far-field are the same.

GAUSSIAN BEAMS

Apodization, or nonuniformity of aperture irradiance, alters diffraction patterns. If pupil irradiance is nonuniform, the formulas and results given previously do not apply. This is important to remember because most laser-based optical systems do not have uniform pupil irradiance. The output beam of a laser operating in the TEM_{00} mode has a smooth Gaussian irradiance profile. Formulas used to determine the focused spot size from such a beam are discussed in *Gaussian Beam Optics*. Furthermore, when dealing with Gaussian beams, the location of the focused spot also departs from that predicted by the paraxial equations given in this chapter. This is also detailed in *Gaussian Beam Optics*.

Energy Distribution in the Diffraction Pattern of a Circular or Slit Aperture

Ring or Band	Circular Aperture Position (x)	Circular Aperture Relative Intensity (I_x/I_0)	Circular Aperture Energy in Ring (%)	Slit Aperture Position (x)	Slit Aperture Relative Intensity (I_x/I_0)	Slit Aperture Energy in Band (%)
Central Maximum	0.0	1.0	83.8	0.0	1.0	90.3
First Dark	1.22π	0.0		1.00π	0.0	
First Bright	1.64π	0.0175	7.2	1.43π	0.0472	4.7
Second Dark	2.23π	0.0		2.00π	0.0	
Second Bright	2.68π	0.0042	2.8	2.46π	0.0165	1.7
Third Dark	3.24π	0.0		3.00π	0.0	
Third Bright	3.70π	0.0016	1.5	3.47π	0.0083	0.8
Fourth Dark	4.24π	0.0		4.00π	0.0	
Fourth Bright	4.71π	0.0008	1.0	4.48π	0.0050	0.5
Fifth Dark	5.24π	0.0		5.00π	0.0	

Note: Position variable (x) is defined in the text.

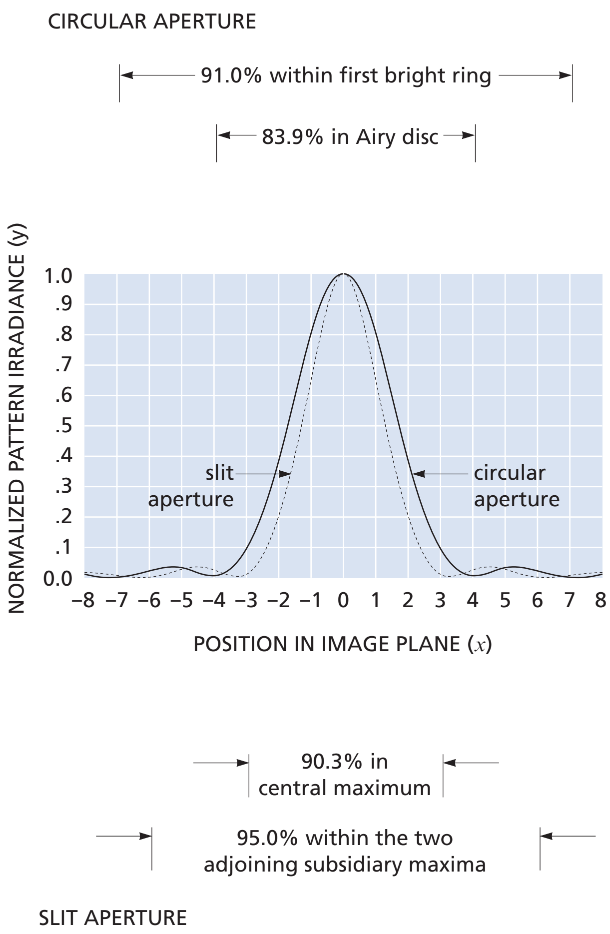

Figure 1.28 **Fraunhofer diffraction pattern of a singlet slit superimposed on the Fraunhofer diffraction pattern of a circular aperture**